Concept: Quadratic Functions

Name:

- **You should have completed Equations – Section 5 Part A: Problem Solving before beginning this handout.**

PART B: COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Graphing > **Quadratic Functions**

NOTE: *Use the* ***Menu*** *button in order to get to the lesson where you left off.*

Work through all Sub Lessons of the following Lessons **in order**:

- *Intercepts of a Quadratic Function*
- *Examples*
- *Maximize Cage Area*
- *Maximize Potato Income*
- *Bob's Beach Ball*
- *Hit the Brakes!*

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

NOTES

1. **Intercepts of a Quadratic Function**

(a) **Method 1:** Finding the intercepts from a ***graph.***

When do the x-intercepts occur? ***when y = 0.***

When do the y-intercepts occur? ***when x = 0.***

(b) Method 2: Finding the intercepts by ***factoring.***

Step1. Set y = ***0.***

Step2. ***Factor*** the right side of the equation.

Step3. Set the factors equal to ***0.***

Step 4. Solve for **x.**

The solutions that we get are actually the points at which the graph cuts the ***x axis.***

(c) **Method 3:** Finding the intercepts using the ***Quadratic*** Formula.

The formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PART B : OFF COMPUTER EXERCISES

1. Given the quadratic function $y = x^2 + 2x - 8$,

(a) Find the x-intercepts by factoring. *(Remember to set y=0)*

To find the x-intercepts, set y = 0.

$$0 = x^2 + 2x - 8$$

Factoring: $0 = (x + 4)(x - 2)$

$\therefore$ $x + 4 = 0$ **or** $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

The x-intercepts are (-4, 0) and (2, 0)

(b) Find the y-intercepts.

To find the y-intercepts, set x = 0.

$$y = x^2 + 2x - 8$$

$$y = (0)^2 + 2(0) - 8$$

$$y = 0 + 0 - 8$$

$$y = -8$$

The y-intercept is (0,-8).

2. Given the quadratic function $x^2 - 9x - 22$,

(a) Find the x-intercepts by using the quadratic formula.

Substitute a = 1, b=-9, c=22 into

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-22)}}{2(1)}$$

$$x = \frac{+9 \pm \sqrt{81 - (-88)}}{2}$$

$$x = \frac{+9 \pm \sqrt{169}}{2}$$

$$x = \frac{+9 \pm 13}{2}$$

$x = \dfrac{9 + 13}{2}$ *or* $x = \dfrac{9 - 13}{2}$

$x = \dfrac{22}{2}$ *or* $x = \dfrac{-4}{2}$

$x = 11$ *or* $x = -2$

The x-intercepts are (11, 0) and (-2, 0).

(b) Find the y-intercepts.

To find the y-intercepts, set x = 0.

$y = x^2 - 9x - 22$

$y = (0)^2 - 9(0) - 22$

$y = -22$

The y-intercept of $y = x^2 - 9x - 22$ ***is (0, -22).***

3. Given the quadratic function $x^2 - 13x + 40$,

(a) find the x-intercepts by factoring.

To find the x-intercepts, set y = 0.

$$0 = x^2 - 13x + 40$$

Factoring $0 = (x - 8)(x - 5)$

$\therefore$ $x - 8 = 0$ **or** $x - 5 = 0$

$x = 8$ $\qquad$ $x = 5$

The x-intercepts of $x^2 - 13x + 40$ are (8, 0) and (5, 0)

(b) find the x-intercepts by using the quadratic formula.

Substitute a = 1, b = -13, c = 40 into

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(40)}}{2(1)}$$

$$x = \frac{13 \pm \sqrt{169 - 160}}{2}$$

$$x = \frac{13 \pm \sqrt{9}}{2}$$

$$x = \frac{13 \pm 3}{2}$$

$x = \frac{13 + 3}{2}$ *or* $x = \frac{13 - 3}{2}$

$x = \frac{16}{2}$ *or* $x = \frac{10}{2}$

$x = 8$ *or* $x = 5$

The x-intercepts are (8, 0) and (5, 0).

4. Fill in the blanks.

(a) $y = -x^2 + 2x + 3$ is a ***quadratic*** function.

(b) Its graph is a ***parabola.***

(c) The curve is concave down because ***x^2 is negative.***

Substitute a = -1, b = 2 into $x = \frac{-b}{2a}$

$= \frac{-2}{2(-1)}$

$= 1$

(d) The x-value of the vertex is ***1.***

The x-value of the vertex can be determined by completing the square

$y = -x^2 + 2x + 3$ ***Completing the square:*** ***$-(x^2 - 2x)$***
$-(x-1)^2 + 1$

Substitute $-(x-1)^2 + 1$ into $y = -x^2 + 2x + 1$

$\therefore$ $y = -(x-1)^2 + 1 + 3$
$= -(x-1)^2 + 4$

The highest that y can be is 4 as the vertex is (1,4).
The x-value of the vertex is 1.

(e) The y-value of the vertex is ***4.***

(f) The axis of symmetry is x = ***1 .***

(g) Find the x-intercepts by factoring.

$y = -x^2 + 2x + 3$

To find the x-intercepts, set y = 0.

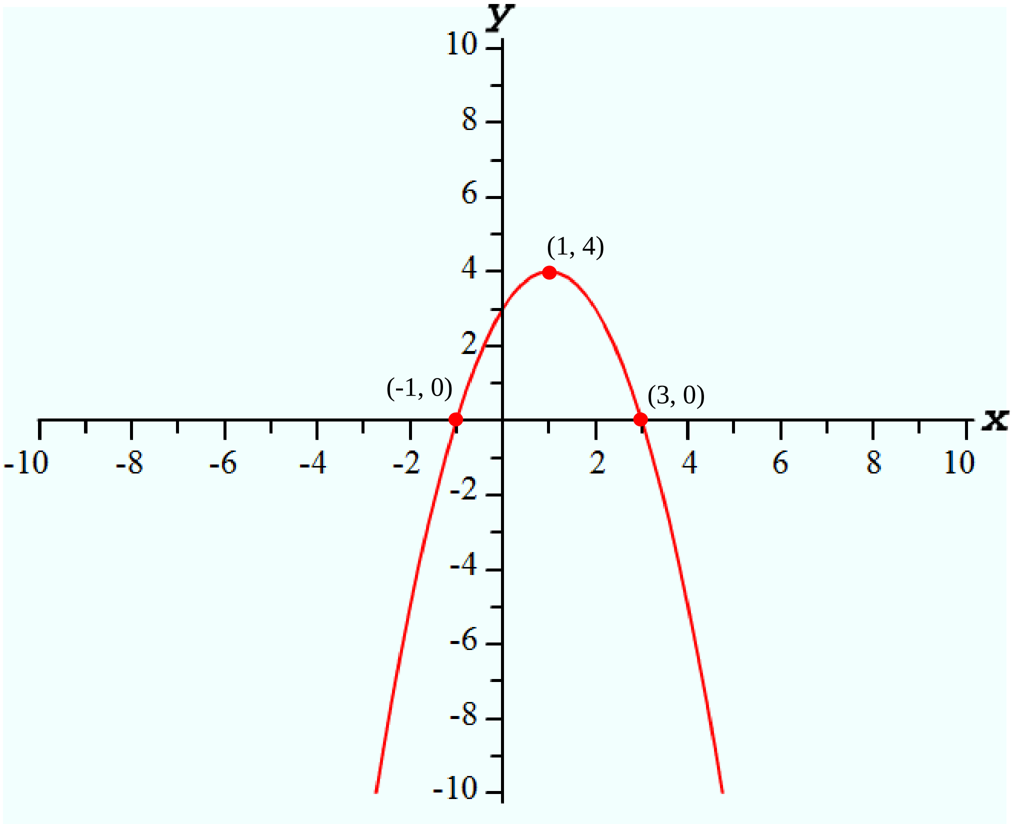
$0 = -x^2 + 2x + 3$

Factor ***$0 = (-x-1)(x-3)$***

So ***$-x - 1 = 0$*** ***or*** ***$x - 3 = 0$***
$x = -1$ ***$x = 3$***

The x-intercepts are (-1, 0) and (3, 0)

(h) Sketch the graph.



5. Fill in the blanks.

(a) $y = x^2 + 10x + 24$ is a ***quadratic*** function.

(b) Its graph is a ***parabola.***

(c) The curve is concave up because ***the coefficient of x^2 is positive.***

(d) The x-value of the vertex is ***-5.***

Substitute a = 1, b = 10 into $x = \frac{-b}{2a}$

$= \frac{-10}{2(1)}$

$= -5$

The x-value of the vertex can be determined by completing the square

$y = x^2 + 10x + 24$

$x^2 + 10x = (x^2 + 10x)$
$= (x+5)^2 - 25$

Substitute $(x+5)^2 - 25$ into $y = x^2 + 10x + 24$

$\therefore$ $y = (x+5)^2 - 1$
$= (x+5)^2 - 1$

Examining $y = (x+5)^2 - 1$
The lowest that y can be is -1 and this occurs when x = -5
$\therefore$ The vertex is (-5,-1)

(e) The y-value of the vertex is **-1.**

(f) The axis of symmetry is x = **-5.**

(g) Find the x-intercepts by factoring.

$y = x^2 + 10x + 24$

To find the x-intercepts, set y = 0.

$$0 = x^2 + 10x + 24$$

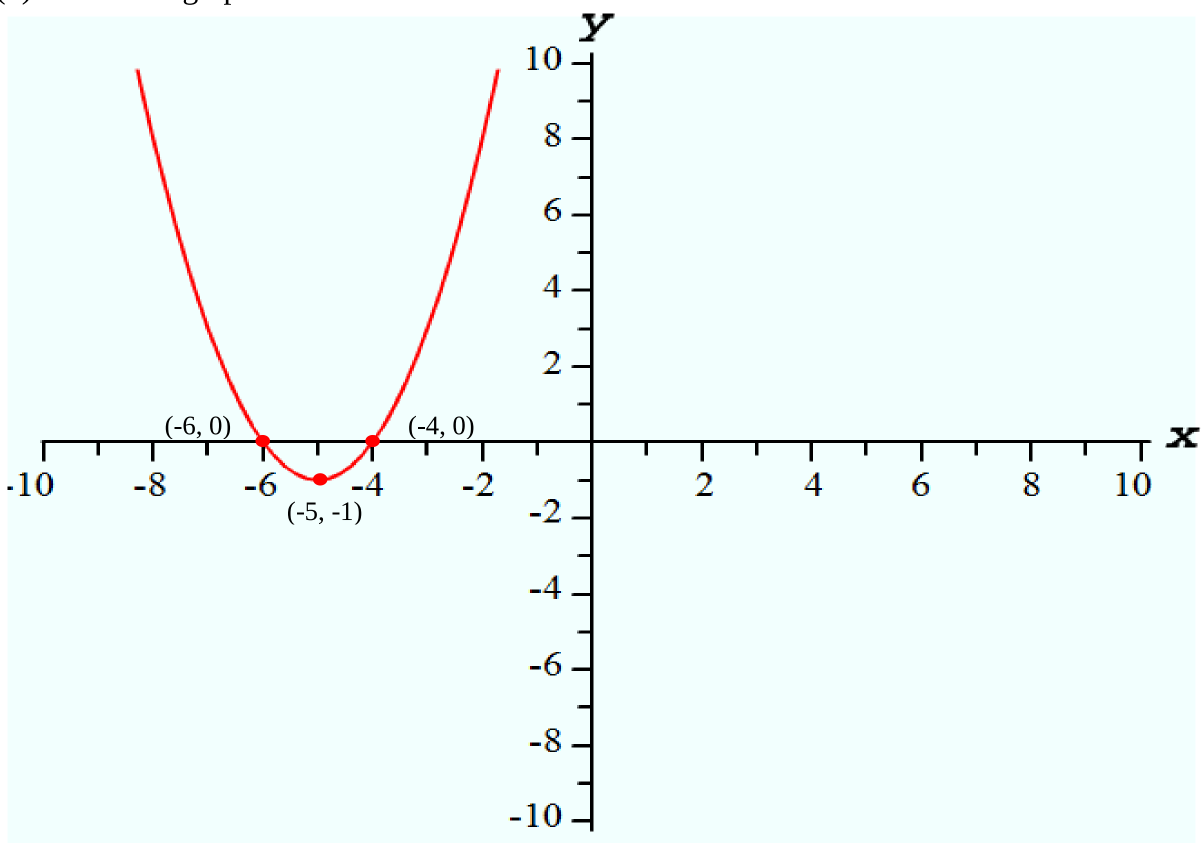
Factoring $\quad 0 = (x+4)(x+6)$

So $\quad x + 4 = 0 \quad$ ***or*** $\quad x + 6 = 0$

$\quad x = -4 \qquad\qquad x = -6$

The x-intercepts are (-1,0) and (-6,0)

(h) Sketch the graph.



6. Farmer George has a cow pasture to enclose. He has 160m of fence. *What dimensions should he make the enclosure so that the cow pasture is covering the maximum amount of area?*

(Remember to figure out how long ½ of the fence is first, then call one of the 4 sides x)

Half the fence is 160 ÷ 2 = 80 m.

Let the width be x   2
∴ length is 80 –x

∴ A = l × w
= x(80-x)
= $-x^2$ +80x
= $-1(x^2 – 80x)$ ***completing the square***
= $-1(x – 40)^2 + 1600$

Maximum area of 1600 occurs when x = 40.
∴ The field is 40m × 40m (a square)

7. Carl tosses a baseball into the air and then catches it.
$y = 48t – 9t^2$ describes the equation of motion of the baseball.
What was the maximum height reached by the ball?

The maximum height of the ball occurs at the vertex. To determine the vertex, express the curve in standard form *($y=ax^2 + b$)*. ***For the given curve $y = -9t^2 + 48t$ this is done by completing the square.***

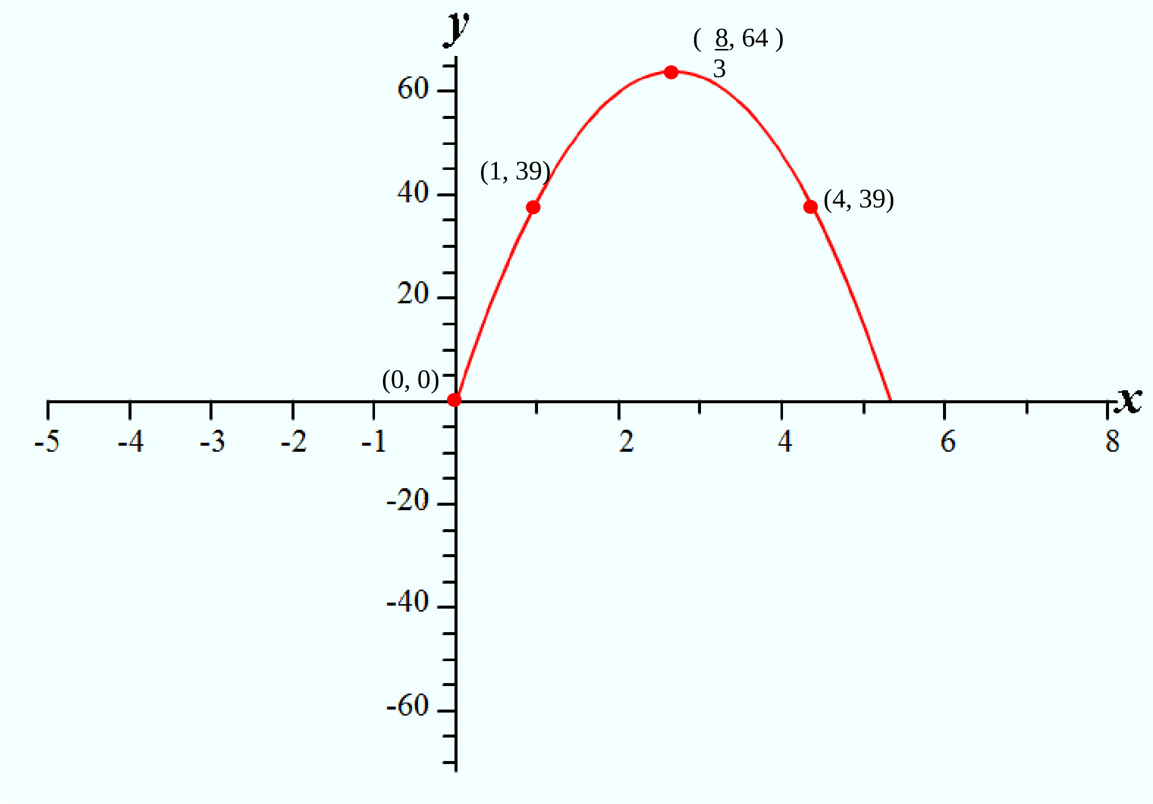
$y = -9t^2 + 48t$ ***Completing the square:*** ***$-(9t^2 - 48t)$***
$-(3t-8)^2 + 64$

Substitute $-(3t-8)^2 + 64$ into $y = -9t^2 + 48t$
∴ $y = -(3t-8)^2 + 64$
The vertex is at (8/3, 64)
∴ The maximum height is 64 units and is reached in 2.6 seconds.

At t=1 $y = -(3-8)^2 + 64$
= 39
At (1, 39) the ball is rising.

At t=4 $y = -(3x4-8)^2 + 64$
=39
At (4, 39) the ball is falling.

At t=0 $y = -(0-8)^2 + 64$
= 0


y
x
60
40
20
-20
-40
-60
-5
-4
-3
-2
-1
2
4
6
8
(8/3, 64)
(1, 39)
(4, 39)
(0, 0)