Concept: Quadratic Functions

Name:

PART A: COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Graphing > **Quadratic Functions**

Work through all Sub Lessons of the following Lessons **in order**:

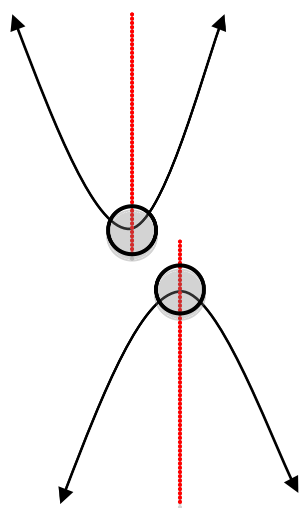
- *Introductory Examples*
- *Definitions*
- *The Role of a*
- The Role of c
- *Completing the Square With Tiles*
- *Completing the Square The Pattern*
- *Completing the Square Examples*
- *Complete the Square to find the Role of B*

NOTE*:* You will not be finishing the entire section before stopping to complete some **OFF COMPUTER EXERCISES**.

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

When you reach the end of the lesson *Complete the Square to find the Role of B* on the computer, move on to the **OFF COMPUTER EXERCISES** below.

NOTES



1. The sketch at the right is ***concept up***.
   Mark the vertex on the sketch.

   The vertex here is a ***minimum*** (maximum or minimum).
   Draw the axis of symmetry on the sketch.

2. The sketch at the right is ***concave down***.
   Mark the vertex on the sketch.

   The vertex here is a ***maximum*** (maximum or minimum).
   Draw the axis of symmetry on the sketch.

3. **Parabola** *(Make notes concerning concave up, concave down, vertex, axis of symmetry.)*

******See previous page***
Parabola is said to be symmetrical about the 'axis of symmetry'.
Axis of symmetry meets the parabola at the vertex.

4. **Quadratic Functions**

The general Quadratic Function is y = $\underline{ax^2 + bx + c}$

A Quadratic Function must always have an $\underline{x^2}$ term.

5. **The role of a:**

- If **a** is positive, the parabola is ***concave up.***
- If **a** is positive and **a** increases, the parabola gets ***narrower*.**
- If **a** is negative, the parabola is ***concave down*.**
- If **a** is negative and **a** decreases, the parabola gets ***narrower*.**

6. **The role of c:**

- In $y = ax^2 + c$: If **c** increases, the parabola shifts ***up.***

  If **c** decreases, the parabola shifts ***down*.**

7. **The role of b:**

- IN GENERAL, for $y = ax^2 + bx + c$ … the x value of the vertex is $\frac{-b}{2a}$

  (use letters for this.)

To find y, we ***substitute*** back into the equation $y = ax^2 + bx + c$.

PART A : OFF COMPUTER EXERCISES

1. Identify a, b and c in the following quadratic functions.

Function	a	b	c
$y = x^2 + 4x + 5$	**1**	**4**	**5**
$y = 2x^2 + 6$	**2**	**0**	**6**
$y = -3x^2 + 7x$	**-3**	**7**	**0**
$y = 5x^2 + x$	**5**	**1**	**0**

2. Summarize (in the chart below) how each of the changes would affect the original graph.

Original graph	Change to ..	Did it affect a, b or c?	How does the original graph change? (i.e. it got narrower, it moved up etc
$y = x^2$	**$y = 2x^2$**	**a**	**- narrower**
$y = x^2$	**$y = x^2 + 5$**	**c**	**- moved up**
$y = x^2$	**$y = x^2 - 4$**	**c**	**- moved down**
$y = x^2 + 1$	**$y = 3x^2 + 1$**	**a**	**- narrower**

3. Find the vertex (ie: what you did in the *Role of b* lesson) for the following:

(a) $x^2 + 4x - 2$
$= [x^2 + 4x] - 2$
$= [x^2 + 4x + \mathbf{4} - \mathbf{4}] - 2$
$= [(x + 4x + 4) - 4] - 2$
$= [(x+2)^2 - 4] - 2$
$= (x+2)^2 - 4 - 2$
$= (x+2)^2 - 6$

The minimum value of $(x+2)^2$ is 0
This happens when $x + 2 = 0$
or when $x = -2$

(b) $x^2 - 6x + 14$
$= [x^2 - 6x] + 14$
$= [x^2 - 6x + \mathbf{9} - \mathbf{9}] + 14$
$= [(x^2 - 6x + 9) - 9] + 14$
$= [(x - 3)^2 - 9] + 14$
$= (x - 3)^2 - 9 + 14$
$= (x - 3)^2 + 5$

The minimum value of $(x-3)^2$ is 0
This happens when $x-3 = 0$
or when $x = 3$

Substitute $x = -2$ into

$y = (x + 2)^2 - 6$
$y = 0^2 - 6$
$y = -6$

∴ ***The vertex is at (-2, -6)***

Substitute $x = -2$ into

$y = (x - 3)^2 + 5$
$y = 0^2 + 5$
$y = 5$

∴ ***The vertex is at (2, 5)***

4. Other than $y = 2x^2 - 3x + 1$, *do any other parabolas contain the points (-1, 6), (0, 1) and (1, 0) ?*

NO!

Solve for the parabola's passing through the three points given.

Use $y = ax^2 + bx + c$

Using (-1,6)
$6 = a(-1^2) + b(-1) + c$
$6 = a - b + c \qquad (1)$

Using (0,1)
$1 = a(0^2) + b(0) + c$
$1 = c \qquad (2)$

Using (1,0)
$0 = a(1^2) + b(1) + c$
$0 = a + b + c \qquad (3)$

From (2) $c = 1$

Substitute c= 1 into (1)
$6 = a - b + c$
$6 = a - b + 1$
-1) $6 - 1 = a - b + 1 - 1$
$5 = a - b \qquad (4)$

Substitute c= 1 into (3)
$0 = a + b + c$
$0 = a + b + 1$
-1) $0 - 1 = a + b + 1 - 1$
$-1 = a + b \qquad (5)$

Add (4) and (5)

$$\begin{array}{rl} 5 &= a - b \qquad (4) \\ + \; -1 &= a + b \qquad (5) \\ \hline 4 &= 2a \end{array}$$

÷2) $\frac{4}{2} = \frac{2a}{2}$

$2 = a$

Substitute a = 2 and c= 1 into (3)

$\therefore$ $0 = a + b + c$
$0 = 2 + b + 1$
$0 = b + 2 + 1$
$0 = b + 3$

-3) $0 - 3 = b + 3 - 3$
$-3 = b$

$\therefore$ ***The only solution is*** $y = 2x^2 - 3x + 1$. ***No other values for a,b, and c resulted from the solution of the 3 equations.***