Concept: Slope of a Line

Name:

Warm Up

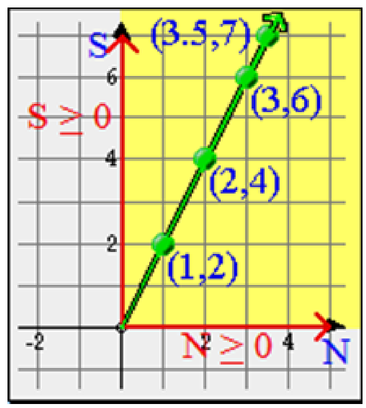
The following suggested activities would serve as a review to consolidate previous learning. While promoting rich mathematical dialog, they will also provide students with the background necessary to be successful in this section.

NOTE: You will need graph paper and graphing calculators for some of these activities.

1. The Elastic Example *(from Topic 6)*

- We let N be the # of washers *(we also included parts of washers).*
- Let S be the length of the stretch of the elastic.
- We calculated the ordered pairs. *(We noticed a pattern in these ordered pairs).*
- From the pattern we were able to write an equation, which was ***S=2N***.
- We should graph ***enough*** points so that we can "see" a ***pattern***.

N	S
1	2
2	4
3	6
3.5	7



NOTE: *Because we can attach* ***parts*** *of washers, we can* ***join*** *the points with a* ***straight*** *line or a* ***continuous*** *curve.*

2. Patterns in Special Relations

Objective: Investigate the roles of $\#_1$ and $\#_2$ in relations of the form $\mathbf{y = \#_1 x + \#_2}$.

Investigation 1:

(a) Investigate the role of $\#_1$ by keeping ***#2*** constant and varying ***#1***.

Graph each of the following on the same axis below:

Clearly identify each line by using different colored pencils or pens.

y = -2x - 3

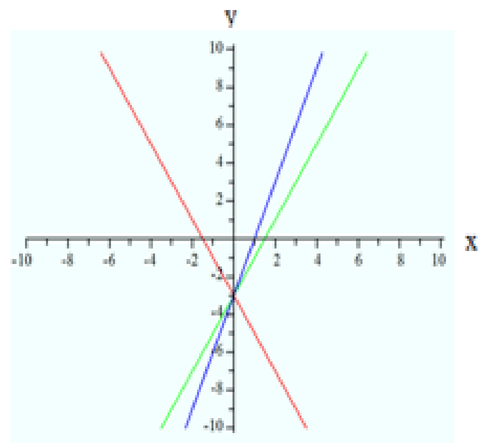
x	y
-2	1
-1	-1
0	-3
1	-5

y = 2x - 3

x	y
-1	-5
0	-3
1	-1
2	1

y =3x - 3

x	y
-1	-6
0	-3
1	0
2	-3



(b) *How are the 3 lines above the same?*

- ***All lines are straight lines.***
- ***All show a linear relationship.***
- ***All lines have the same y-intercept (0, -3)***

(c) *How are the 3 lines different?* ***The lines have different slopes.***

(d) *What seems to be the role of* $\#_1$ *in an equation of the form* $y = \#_1 x + \#_2$ *?*

$\#_1$ in the equation is the slope of the line

Investigation 2:

(a) Investigate the role of $\#_2$ by keeping ***$\#_1$*** constant and varying ***$\#_2$***.

Graph each of the following on the same axis below:

Clearly identify each line by using different colored pencils or pens.

y = 2x + 1

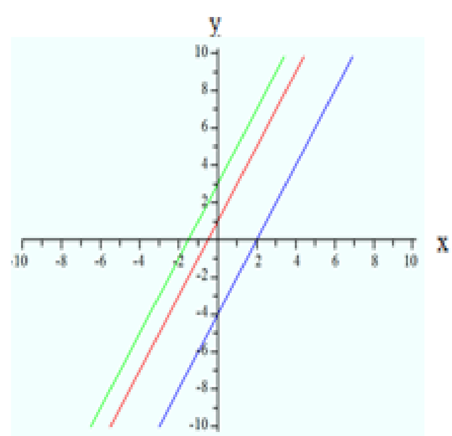
x	y
-1	-1
0	1
1	3
2	5

y = 2x + 3

x	y
-2	-1
-1	1
0	3
1	5

y =2x - 4

x	y
0	-4
1	-2
2	0
3	2



(b) *How are the 3 lines above the same?* ***All lines are straight lines and have the same slope.***

(c) *How are the 3 lines different?* ***The y-intercept is different. (0, 1) (0, 3) (0. -4)***

(d) *What seems to be the role of* $\#_2$ *in an equation of the form* $y = \#_1 x + \#_2$? Discuss your findings with a classmate.

$\#_2$ in the equation is the y-intercept.

3. <u>Graph and Interpret *Bungee Jumping*</u>

You are *Bungee Jumping…*

The equation, which enables you to find your height above ground level at any time from 0 to 8 seconds after the cord first stretches, is ... $\mathbf{y = x^2 - 6x + 8}$

Let x represent … time in seconds after the cord first begins to stretch.

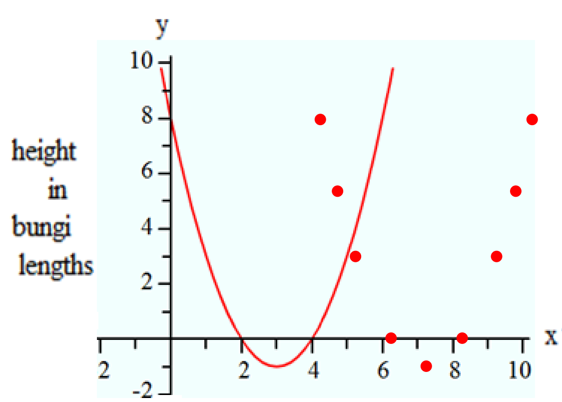
Let y represent… the person's height (in bungee lengths) above the ground.

Instructions:

- Choose enough values for x between 0 and 8 inclusive to give you enough of a Pattern, so you can draw the graph as smoothly as possible.
- Complete the chart.
- Draw the graph as accurately as possible.

We can join the points because ***<u>the time (x) is continuous</u>***.

x	y
0	**8**
.5	**5.25**
1	**3**
2	**0**
3	**-1**
4	**0**
5	**3**
6	**8**
7	**15**
8	**24**



Interpret the Graph:

(a) *How far above ground are you when the cord begins to stretch?*

The cord begins to stretch at 8 bungee lengths above the ground.

(b) Will we need to dig a hole in the ground below you? If so, how deep must the hole be?

Yes, it is necessary to dig a hole. From 2-4 seconds the jumper is below ground level. At 3 seconds the jumper is 1 bungee length below the ground (-1). The hole will need to be more than 1 bungee length below ground level to ensure the person does not smack the ground.

(c) You will be at the same height at .5 seconds and at **5.5** seconds.

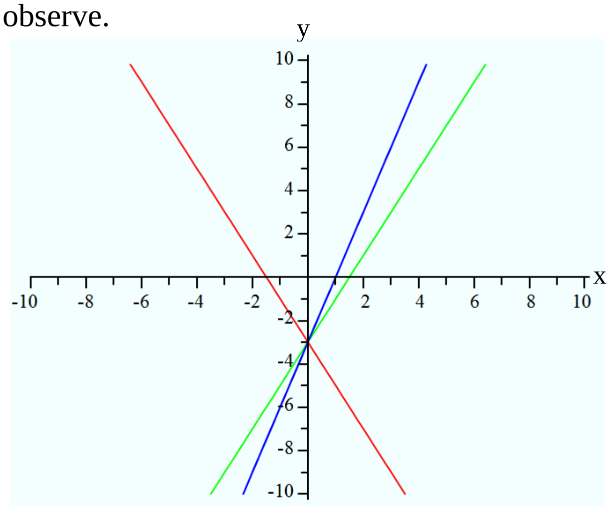
When done, discuss results with a classmate.

4. **An Investigation Activity using a Graphing Calculator**

Your teacher may need to introduce you to a graphing calculator.

NOTE: This will enable you to extend some of the investigations, which you may have done earlier.

In each case, command the graphing calculator to graph each of the following equations. Note and record any patterns that you observe.



Investigation 1: y = -2x - 3

y = 2x – 3

y = 3x – 3

Observations:

- ***The line y = -2x – 3 goes down from left to right.***
  y = 2x – 3 and y = -3x -3 go up from left to right.
- ***Increasing from 2x to 3x results in a steeper slope.***
- ***The y-intercept is (0, -3) for all three graphs.***

Now, use the calculator to graph the equations, which you graphed earlier on this worksheet. Try your own equations like the ones above. *Do you notice patterns? Role of #1 , #2 ?*

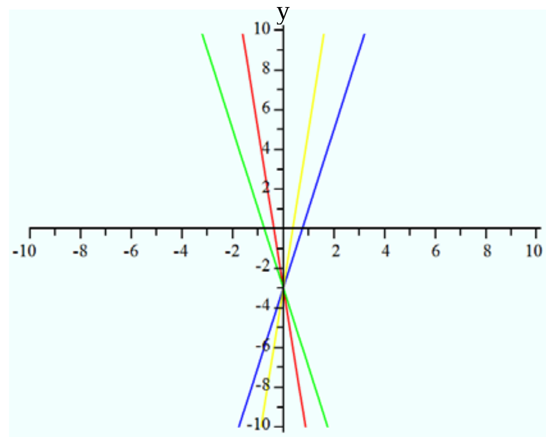
Observations:

- ***#1 in the equations is the slope of the line.***
- ***The lines y = 2x-3 and y = 3x-3 have a positive slope and go up from left to right.***
- ***y = -2x-3 has a negative slope and goes down from left to right.***
- ***The y-intercept for all the lines is (0, -3). #2 is the y-intercept.***

Now, (if your calculator permits).
Graph equations of the form **y = nx - 3** where **n** ranges from -8 to 8

Observations: ***n = -8*** ***n = 4***

n = -4 ***n = 8***



- ***As n increases from -8 to 0 the lines slope less and less down and to the right, at n=0 the line is horizontal.***
- ***As n increases from 0 to 8 the lines slope up from left to right with increasing steepness.***

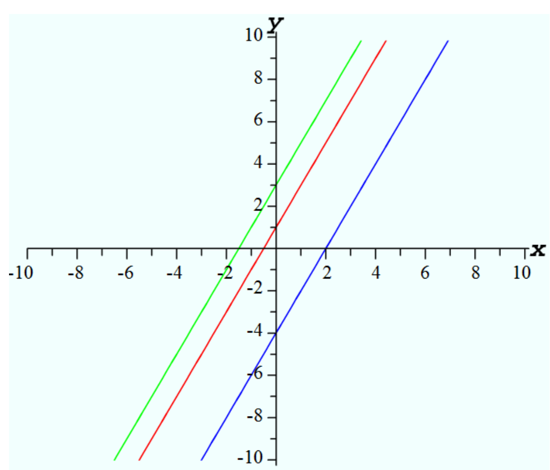
Investigation 2: y = 2x + 1
y = 2x + 3
y = 2x – 4

Now, try your own equations like the ones above. *Do you notice patterns? Role of #1, #2 ?*
Observations:

y = 2x + 1

y = 2x + 3

y = 2x – 4



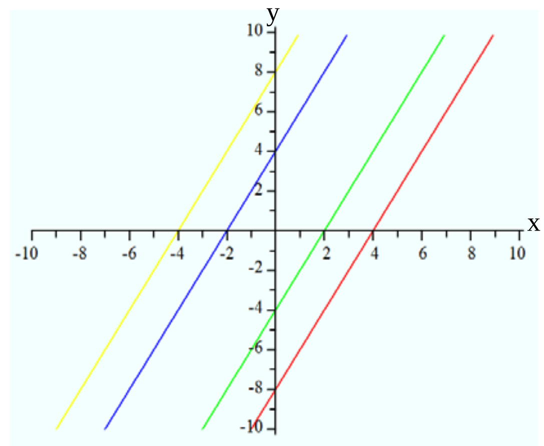
- ***The slope of the lines (Role of #1) is the same and equals 2.***
- ***The y-intercepts [(0,1),(0,3),(0,-4)] are determined by #2 (1,3,-4).***

Now, (if your calculator permits) …
Graph equations of the form **y = 2x + n** where **n** ranges from -8 to 8.

Observations:

n = -8 ■ n = 4 ■

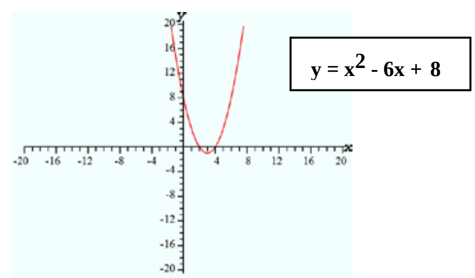
n = -4 ■ n = 8 ■



- ***As n increased from -8 to 8 the y-intercept increases, that is, the lines move up.***

Investigation 3: **$y = x^2 - 6x + 8$** *(the Bungee Jumping example, which you did on paper before)*

Now, try more equations like the one above.



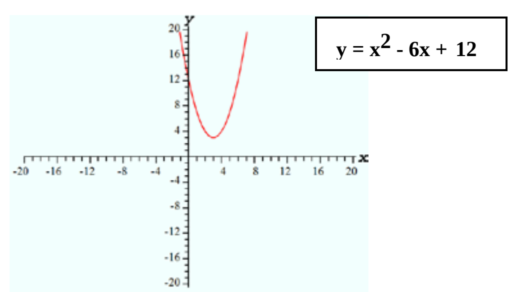
Observations: ***(Answers will vary)***

Examples are as follows:

$y = x^2 - 6x + 8$ ***original***

- ***With the constant equal to 8 the bungee jumper has to worry about hitting the ground and will need a hole to bungee into.***

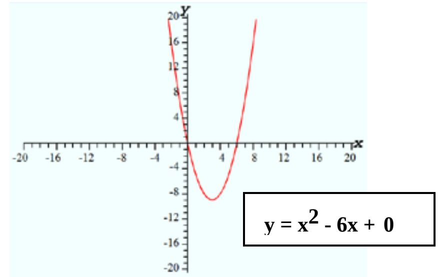
$y = x^2 - 6x + 12$



- ***With the constant equal to 12 the bungee jumper is jumping from a higher height and does not have to worry about hitting the ground.***

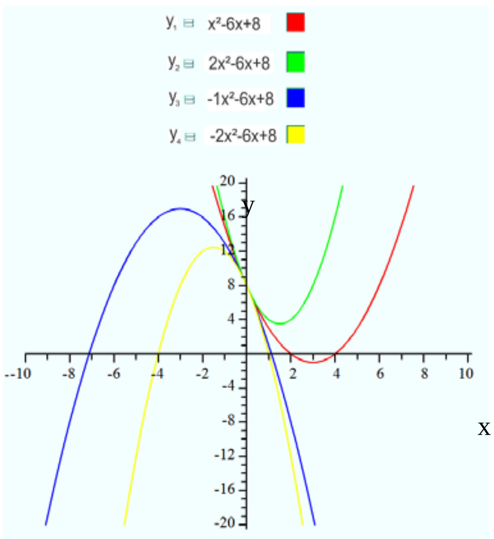
$$y = x^2 - 6x + 0$$

- ***With the constant equal to 0 the bungee jumper is jumping starting at ground level and cannot jump.***



Now (if your calculator permits) …
Replace each of the constants with **n** and vary **n** as you wish.

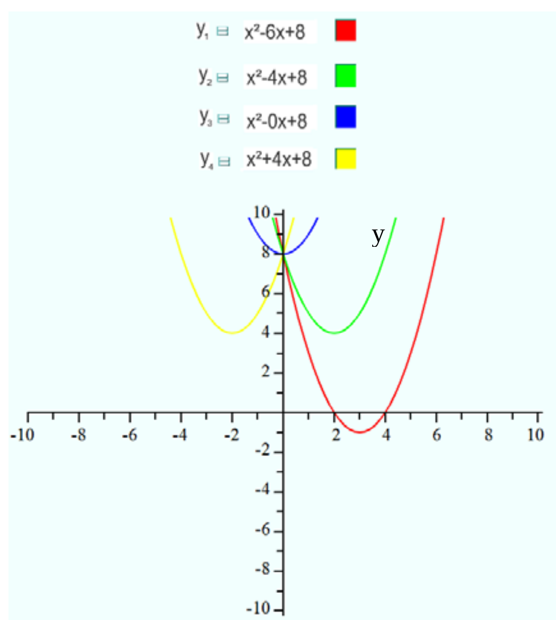
<u>Observations:</u>



x

- ***When the coefficient of x^2 is positive the graph is concave up. When the coefficient of x^2 is negative the graph is concave down.***

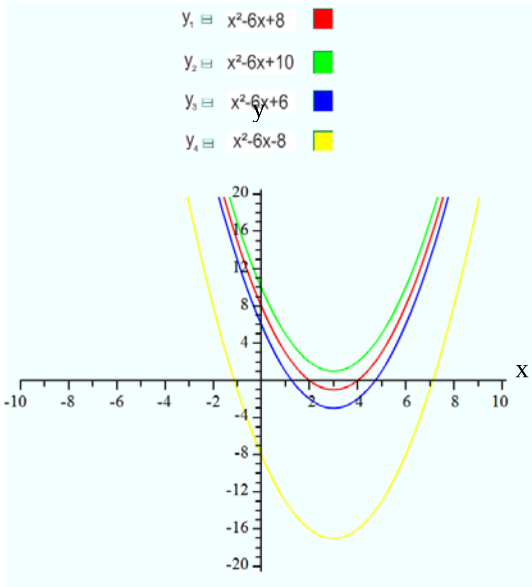
- ***The absolute value of the coefficient of x^2 effects the steepness of the parabola, the larger it is, the steeper the parabola.***



x

- **The value of the coefficient of x in these examples effects the location of the vertex of the parabola.**

- **Each parabola in this example has the same shape.**



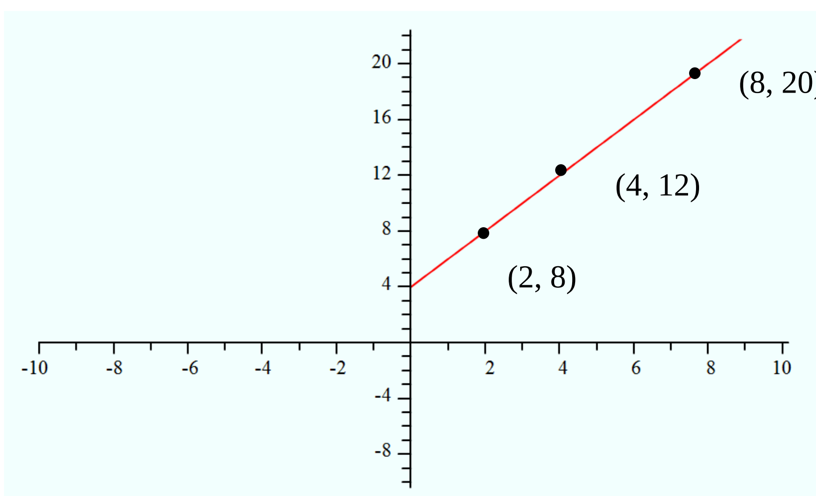
The value of the constant in these equations (-8, 6, 8, 10) effects the height of the vertex.

Each parabola in this example has the same shape

5. **Graph and Interpret (A)**

The cost C, in dollars of replicating CDs is given by the equation **C = 2n + 4** where $4 is the fixed setup cost for replicating *(includes setup of the replicator.. a fixed cost)* and **n** is the number of CDs replicated. *The equation only applies if 1 to 10 CDs are replicated.*

(a) Choose values of the variables and draw a graph of Cost vs. Number of CDs. *(Use Graph Paper)*



n	C
5	**14**
4	**12**
3	**10**
2	**8**
1	**6**
0	**4**

(Notice that negative CD's cannot be copied)

(b) What would it cost to replicate 8 CDs? (Use both the equation and your accurate graph to give you the answer)

If n = 8 then $C = 2n + 4$
$= 2(8) + 4$
$= 16 + 4$
$= 20$

See above graph point (8,20)

It would cost <u>$20</u> to replicate 8 CD's.

c) How many CDs can you replicate with $12? *(Use both the equation and your accurate graph to give you the answer)*

If C = 12 then $12 = 2n + 4$
$12 - 4 = 2n + 4 - 4$
$8 = 2n$
$\frac{8}{2} = \frac{2n}{2}$
$4 = n$

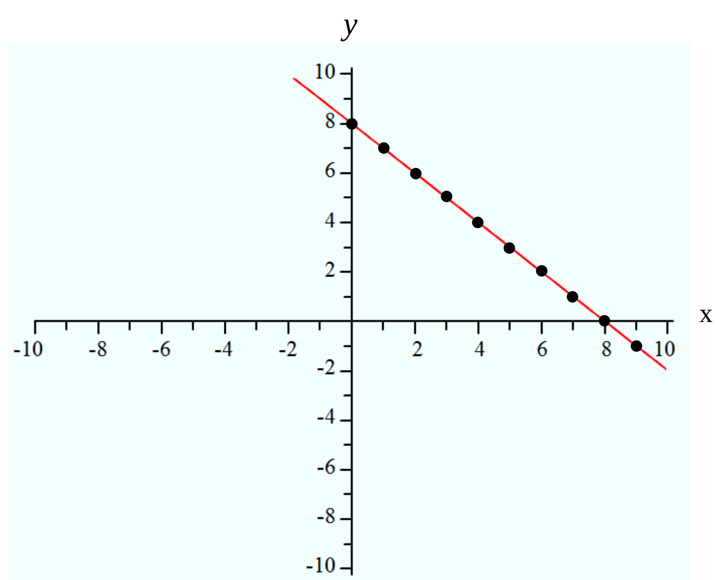
See above graph point (4,12)

For $12, <u>4 CD's</u> could be replicated.

6. **Graph and Interpret (B)**

- Graph the equation **x + y = 8** where **x** and **y** both represent real numbers.
- Show a table of values *(pick enough points to see a pattern).*

x	*y*
-1	*9*
0	*8*
1	*7*
2	*6*
3	*5*
4	*4*
5	*3*
6	*2*
8	*0*



- Graph the ordered pairs which satisfy x **+ y = 8** in an accurate graph.
- *Interpret the graph.*
  Any point on the graph is such that the ***<u>sum</u>*** of ***<u>two</u>*** numbers is always ***<u>8</u>***. As x increases, then ***<u>y decreases</u>*** .

PART A: COMPUTER COMPONENT

In order for you to do the following work, your teacher may need to discuss a way of finding the length of a third side of a right triangle when the lengths of 2 of the sides are given. (Pythagorean Theorem)

Instructions: In UMATH X follow the **Content Menu** path:

Graphing > **Slope of a Line**

Work through all Sub Lessons of the following Lessons **in order:**
- *Introduction to Slope*
- *Slope*
- *Introductory Examples*

NOTE: You will not be finishing the entire section before stopping to complete some **OFF COMPUTER EXERCISES**.

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

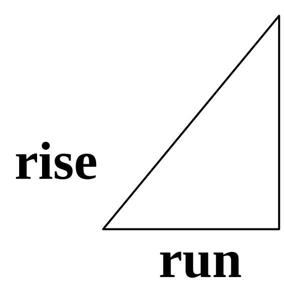
When you reach the end of the lesson *Introductory Examples* on the computer, move on to the **OFF COMPUTER EXERCISES** below.

NOTES

1. Fill in the following using information from the section ***Slope***

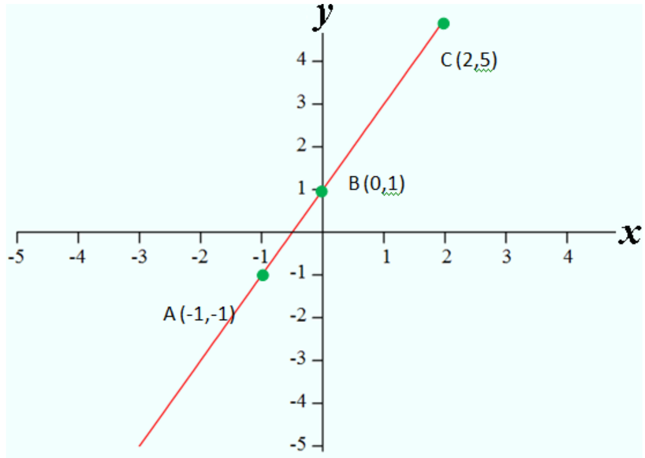
(a) Draw a right triangle and label **rise** and **run** on the triangle.

Slope = $\dfrac{\textbf{rise}}{\textbf{run}}$



(b) Record ***Example 4 of the Introductory Examples*** below.

Place your x axis and your y axis in a convenient place. *Label them.*



Any ***2*** points on a ***<u>straight</u>*** line can be used to find the ***<u>slope</u>*** of that line.

OFF COMPUTER EXERCISES

1. Use your ruler to choose any 3 points with integer coordinates on the graph above. Label the points A, B, C. The points must be in a straight line. The line should be non-horizontal and also non-vertical.

(a) I chose the points A (***-1*** , ***-1***) B (***0*** ,***1***) and C (**2,5**)

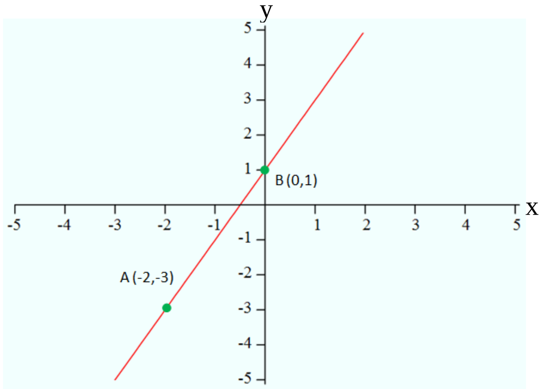
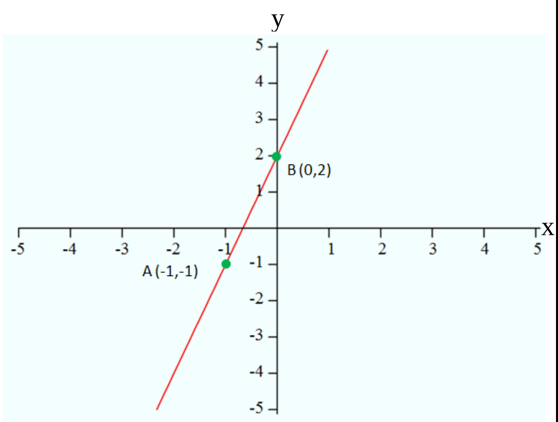
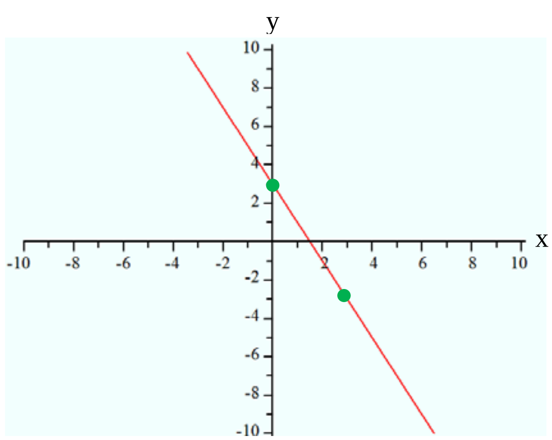
(b) Calculate the slope of line segment AB $= \frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{2}}{\textbf{1}}$

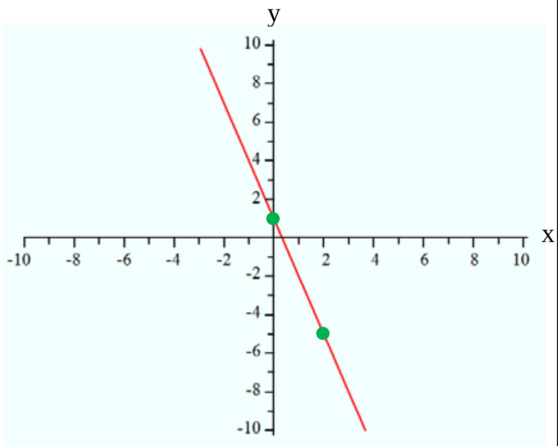
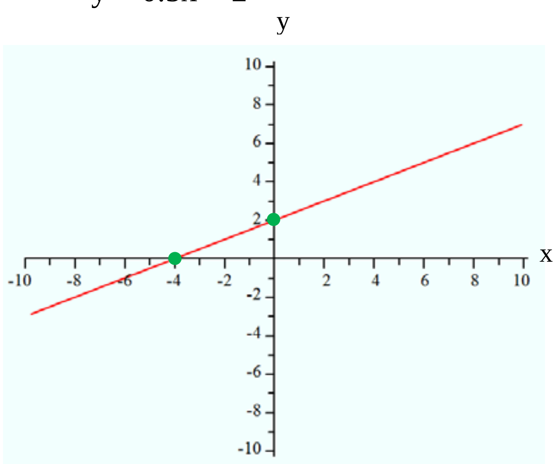
(c) Calculate the slope of line segment BC $= \frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{4}}{\textbf{2}} = \frac{\textbf{2}}{\textbf{1}}$

<u>Conclusion:</u> ***Any 2 points on a straight line can be used to find the slope of that line.***

2. For each of the following equations of relations *(all of the equations result in graphs which are lines)*

- Present a chart to show your ordered pairs which satisfy the equation.
- Use grid paper to graph the points on an xy coordinate grid.
- Use any 2 points on the graph of the line to find its slope.
- Enter the requested information in the chart below.

Equation of line	Points A and B	Slope of Line AB
y = 2x – 4 	**Point A (-2, -3)** **Point B (0, 1)**	**Slope = $\frac{rise}{run}$** **= $\frac{4}{2}$** **= $\frac{2}{1}$** **= 2**
y = 3x + 2 	**Point A (-1, -1)** **Point B (0, 2)**	**Slope = $\frac{rise}{run}$** **= $\frac{3}{1}$**
y = -2x + 3 	**Point A (0, 3)** **Point B (3, -3)**	**Slope = $\frac{rise}{run}$** **= $\frac{6}{-3}$** **= $\frac{-2}{1}$** **= 2**

$y = -3x + 1$ 	**Point A (2, -5)** **Point B (0, 1)**	**Slope** $= \frac{\textbf{rise}}{\textbf{run}} = \frac{6}{-2} = -\frac{3}{1} = -3$
$y = 0.5x + 2$ 	**Point A (-4, 0)** **Point B (0, 2)**	**Slope** $= \frac{\textbf{rise}}{\textbf{run}} = \frac{2}{-4} = -\frac{1}{2}$

I noticed the following pattern...

- ***The difference in the y coordinates is the rise.***
- ***The difference in the x coordinates is the run.***