Concept: Pythagorean Theorem

Name:

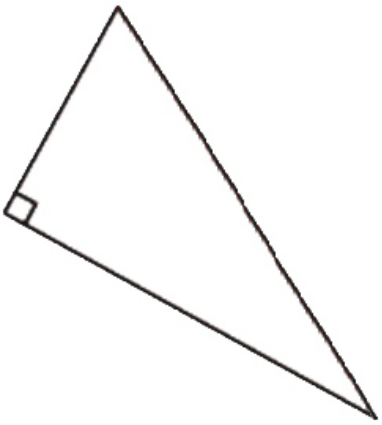
Interesting Fact:

"The Pythagorean Theorem was one of the earliest theorems known to ancient civilizations. This famous theorem is named for the Greek mathematician and philosopher, Pythagoras. Pythagoras founded the Pythagorean School of Mathematics … in southern Italy."

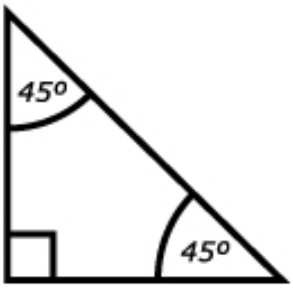
http://jwilson.coe.uga.edu/EMT669/Student.Folders/Morris.Stephanie/EMT.669/Essay.1/Pythagorean.html

Warm Up

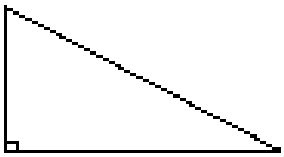
Review right angle triangles. Fill in the blanks.



- A right angled triangle is a polygon with:
  - i. ***3*** corners or vertices
  - ii. ***3*** sides or edges
  - iii. ***1*** internal angle that is a ***right*** angle measuring ***90*** degrees.



- Isosceles right angle triangles
  - i. ***1*** right angle
  - ii. ***2*** equal sides
  - iii. ***2*** other ***acute*** angles always ***45*** degrees



- Scalene right angle triangles
  - i. ***1*** right angle
  - ii. ***no*** equal sides
  - iii. ***2*** other acute angles

COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Exponents > **Pythagorean Theorem**

💻 Work through all Sub Lessons of the following Lessons **in order**:

- *In This Topic*
- *The Right Triangle*
- *Math or Magic?*
- *Squares on a Grid*
- *Squares on the Sides of a Right Triangle*
- *The Pythagorean Theorem*
- *Example Questions*

Additional Required Materials: *long string*
ruler
marker
3 pins
scissors
protractor
pegboard or corrugated cardboard

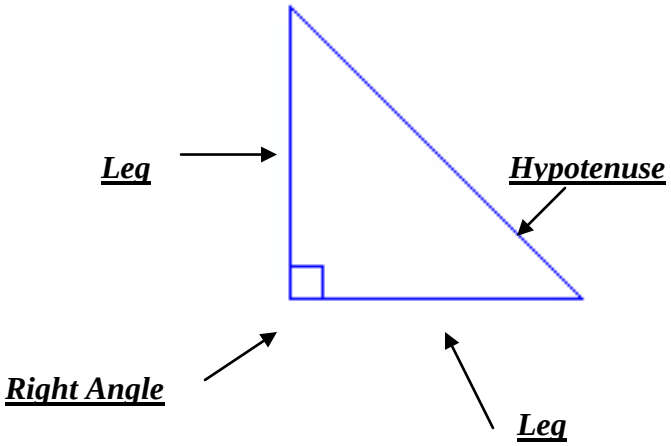
✎ As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

NOTES

Fill in the blanks.

- A right triangle is a triangle with one ***right*** angle.
- A right angle is a 90^0 angle.
- The symbol for a right angle is _____.
- The hypotenuse is the side ***opposite*** the ***right*** angle.
- The other two sides of a right angle triangle are called ***two legs.***

A Right Triangle (Fill in the blanks.)



Omar's Rope Tricks (*Answers will vary)*

Required Equipment:

- a long string - scissors
- a ruler - a protractor
- a marker - corrugated cardboard or pegboard
- 3 pins (12" by 6" or 300mm by 150mm)

For Omar's Rope Trick #1

Instructions:

- make 13 equally spaced markings on the string (perhaps 1" between each mark)
- on your cardboard or pegboard, use pins and place the pins (stakes) where Omar placed them (knot 4, knot 8, knot 13)
- sketch the diagram of the triangle from Omar's Rope Trick #1
- record the lengths of the sides
- use a protractor to measure all three angles of the triangle
- record the degrees of each angle on the sketch

For Omar's Rope Trick #2

Instructions:

- make 31 equally spaced markings on the string
- on your cardboard or pegboard, use pins and place the pins (stakes) where Omar placed them (knot 13, knot 18, knot 31)
- sketch the diagram of the triangle from Omar's Rope Trick #2
- record the lengths of the sides
- use a protractor to measure all three angles of the triangle
- record the angle measures on the sketch

For your own Rope Trick

Instructions:

- think of a "knot number"
- create a string with your number of knots
- try to form a right triangle using all possible combinations of sides
- record your results in the chart

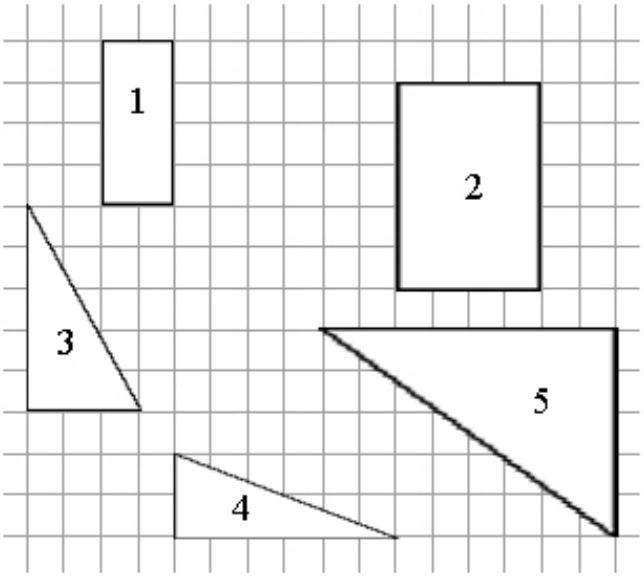
Example:

# of Knots	Possible Triangles Sides	Sketch	Right Angle?
4	**1, 1, 1**		**No**
5	**1,1,2**		**No**

# of Knots	Possible Triangles Sides	Sketch	Right Angle?

Squares on a Grid

1. Find the areas below**.**



(a) Area 1: ***8 sq. units***

(b) Area 2: ***20 sq. units***

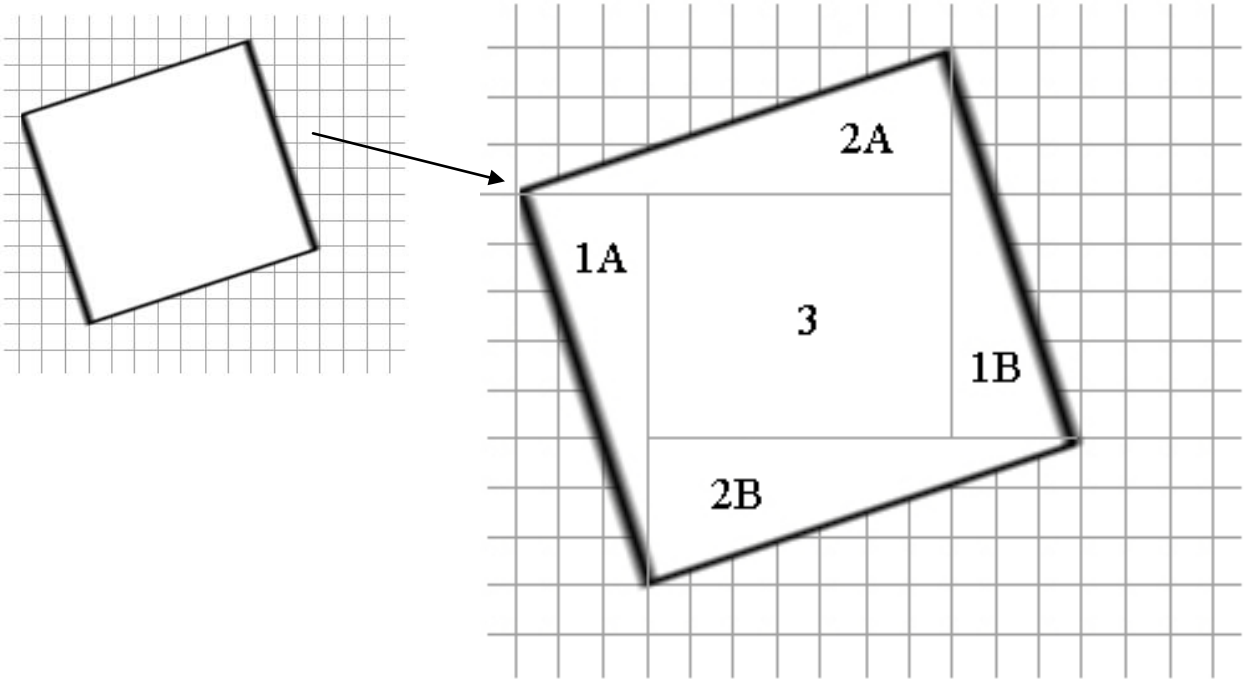
(c) Area 3: ***7.5 sq. units***

(d) Area 4: ***12 sq. units***

(e) Area 5: ***20 sq. units***

Examples 3 and 4 in "Squares on a Grid", shows you how to find the area of a square or rectangle with sides not along the grid lines. Lines are drawn to cut the figure into shapes with areas that can easily be found. Combining triangles make rectangles where *Area = l × w* (*An easier calculation to figure out*).

Combine the triangles and calculate the areas of the rectangles. Then calculate the area for the whole shape.



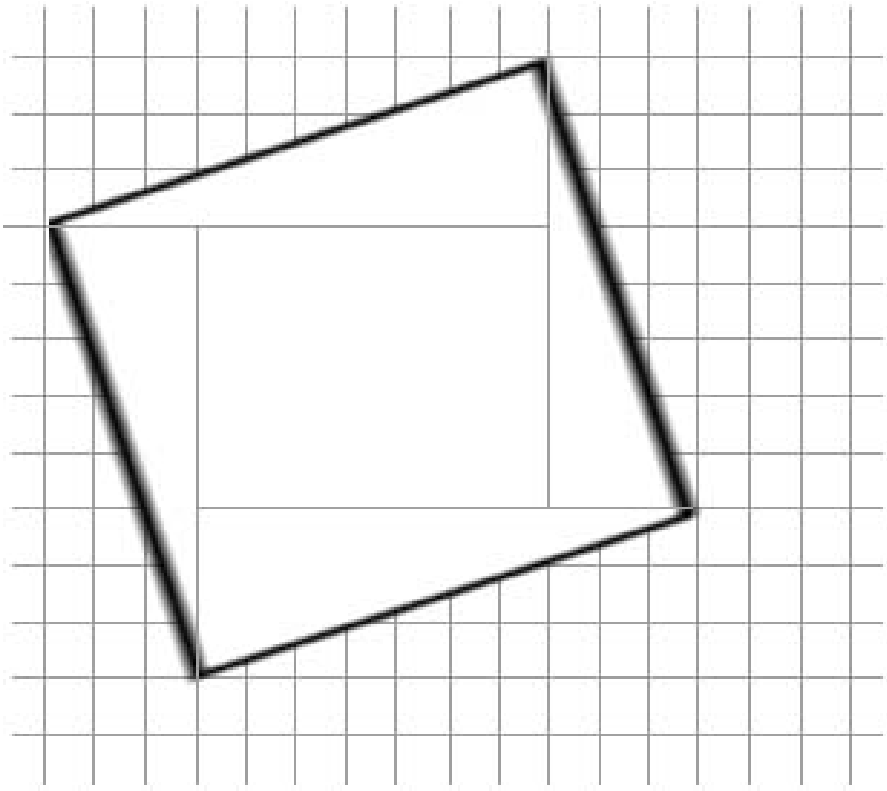
Area of Triangle 1A and 1B = ***24 sq. units***

Area of Triangle 2A and 2B = ***30 sq. units***

Area of Rectangle 3 = ***35 sq. units***

Total Area = ***89 sq. units***

For the following square, complete the questions below:



(a) What is the area of each right triangle? ***15 sq. units***

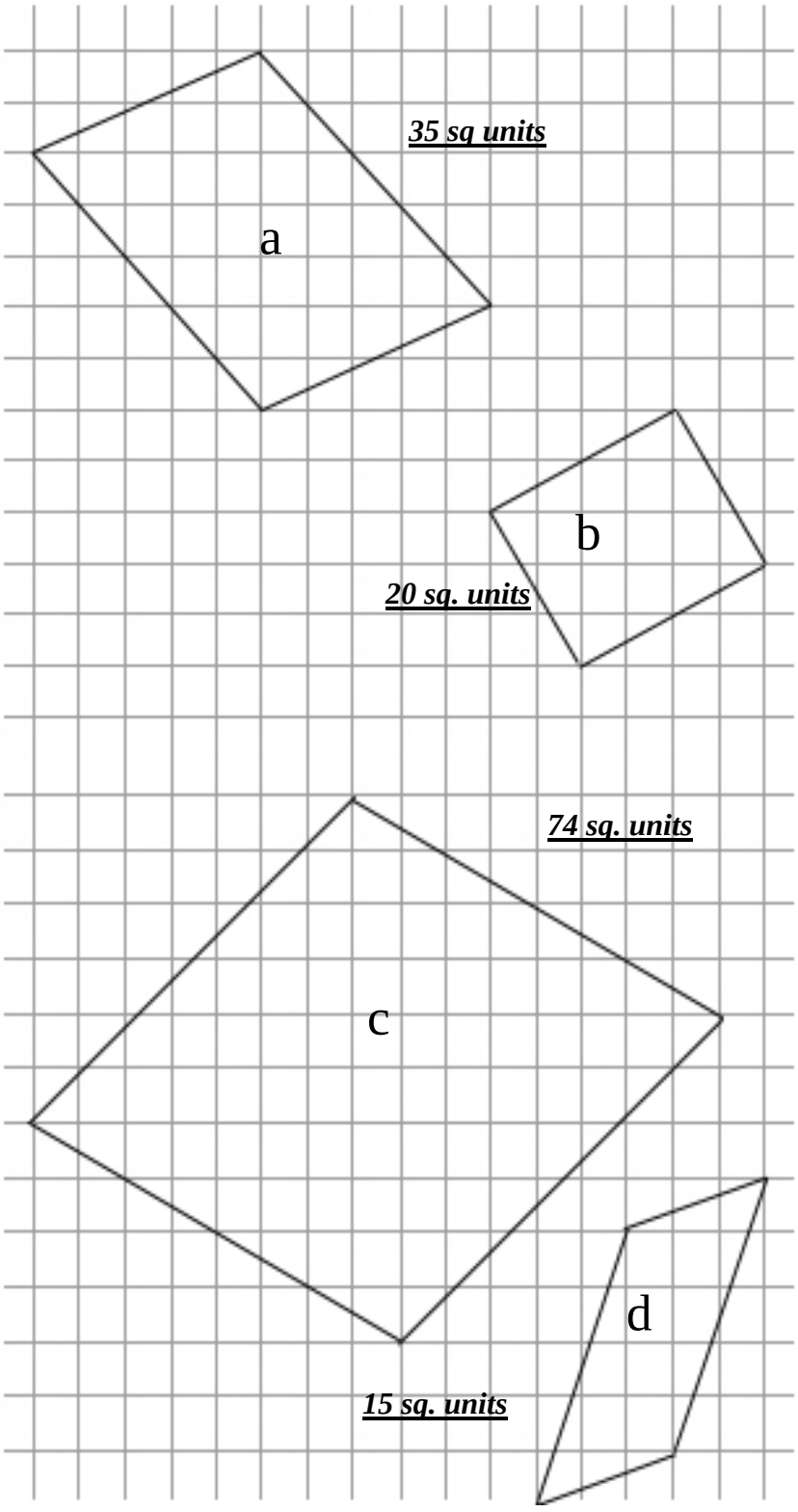
(b) What is the area of each inner square? ***35 sq. units***

(c) What is the area of each original square? ***95 sq. units***

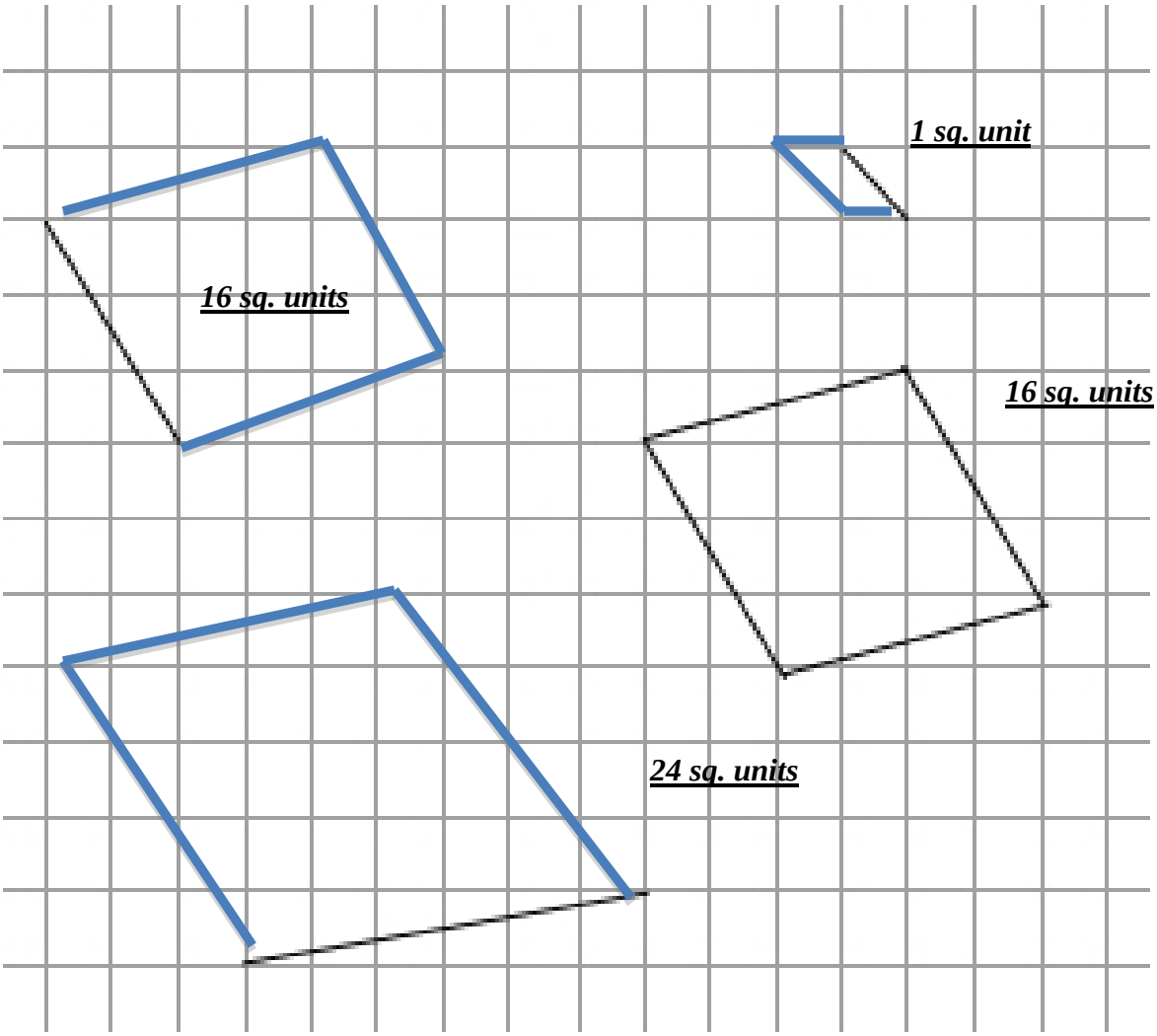
(d) What must be the length of each side of the original square? ***10 sq. units***

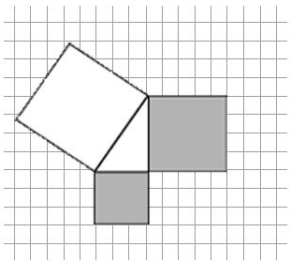
Manipulation of Shapes in Discovering the Pythagorean Theorem

(a) Calculate the area of each of the following figures. Draw lines or use scissors and cut each figure into shapes of which the area can easily be found. ***(line responses will vary)***



(b) Build a square on each line and calculate the area of the squares. The square for line c has been outlined for you.





The Pythagorean Theorem

Fill in the empty boxes with words relating the area of the vertical leg, horizontal leg, and the hypotenuse.

Area of square on one leg	+	***Area of square on on leg***	=	***Area of square on the hypotenuse.***

In general,

- **If one leg has length a, then the area of the square on that leg is a^2.**

- **If one leg has length b, then the area of the square on that leg is b^2**

- **If the hypotenuse has length c, then the area of the square on that leg is c^2**

The Pythagorean Theorem

For any right triangle, $a2 + b2 = c^2$

OFF COMPUTER EXERCISES

1. Use the Pythagorean Theorem to fill in the table below.

   - triangle 1: vertical leg = 5, horizontal leg = 2
   - triangle 2: vertical leg = 5, horizontal leg = 3
   - triangle 3: vertical leg = 4, horizontal leg = 6

Triangle #	Area of Square on Vertical Leg	Area of Square on Horizontal Leg	Area of Square on Hypotenuse
1	**25**	**4**	**29**
2	**25**	**9**	**34**
3	**16**	**36**	**52**

2. A complete answer for the following questions should include diagrams. *You may wish to review the **Example Questions** on the computer.*

   (a) A 9 m post is stood up so that it meets the ground at right angles. A wire is strung from the top of the post to a peg on the ground 7 m from the base of the post. *How long (to 2 decimal places) must the wire be?*

   $9^2 + 7^2 = 81 + 49 = 130 = \sqrt{130} = 11.4m$

   The wire must be 11.4 m long.

   (b) A 4 m ladder is placed 1 m from the base of the building. *How far up the building does the ladder reach?*

   $16 - 1 = 15 = \sqrt{15} = 3.87m$

   The ladder reaches 3.87m up the building.

(c) A, B, C are corners of a rectangular field. AC is a diagonal. If it takes 10 steps to go from A to B and 9 steps to go from B to C, *how many steps could you save by walking directly from A to C?*

$10^2 + 9^2 = 181 = \sqrt{181} = 13.45$

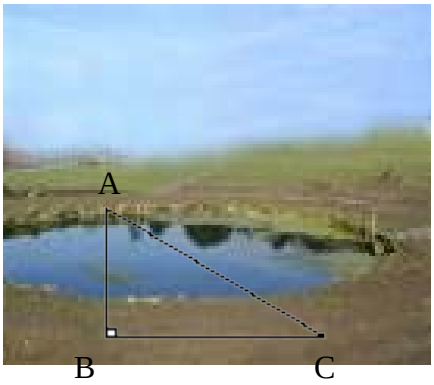
19-13.45= 5.55

You can save almost 6 steps by walking directly from A to C.

3. To find the length of a lake, a surveyor places flags at both ends of the lake. She then walks to another point C such that <ABC = 90^0. She measures and finds the distance from A to C to be 112 m and the distance from B to C to be 91 m. *Find the length of the lake*.

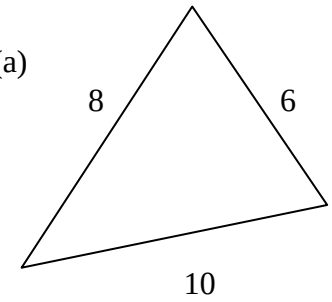
12544 – 8281=4263 = $\sqrt{4263}$ **= 65.29m**

The length of the lake is 65.29m



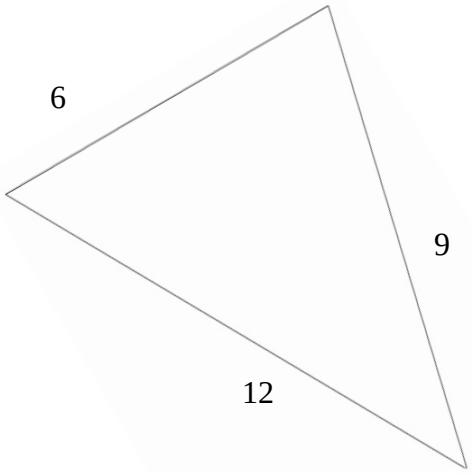
4. Use the Pythagorean relationship to determine if each of the triangles below is a right triangle.

(a)



This is a right triangle because- $8^2 + 6^2 = 10^2$

(b)



This is not a right **because** $6^2 + 9^2 = 12^2$

5. Pythagorean triples are three numbers that satisfy the Pythagorean relationship.

Example: 3, 4, 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$.

(a) 6, 8, 10 ***is a Pythagorean triple because*** $6^2 + 8^2 = 10^2$

(b) 6, 10, 12 ***is not a Pythagorean triple because*** $6^2 + 10^2$ ***does not*** **=** 12^2

(c) 9, 12, 15 ***is a Pythagorean triple because*** $9^2 + 12^2 = 15^2$

(d) 5, 8, 10 ***is not a Pythagorean triple because*** $5^2 + 8^2$ ***does not*** **=** 10^2

(e) 12, 16, 20 ***is a Pythagorean triple because*** $12^2 + 16^2 = 20^2$