Concept: Solving Absolute Value Equations

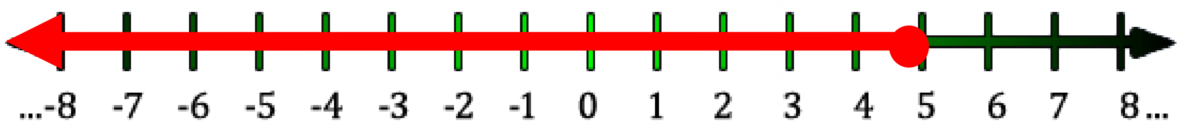
Name:

Warm Up

1. Determine what values of x make each inequality true. *Graph each answer.*

(a) $9x - 2 \leq 7x + 8$

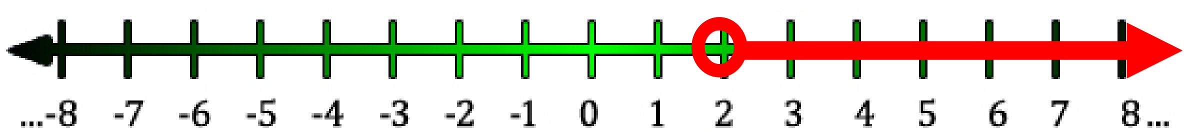
	$9x - 2 \leq 7x + 8$
-7x)	$2x - 2 \leq 8$
+2)	$2x \leq 10$
÷2)	$x \leq 5$



> *Remember:*
>
> *If you multiply or divide both sides by a negative quantity, the inequality sign must be* ***reversed***.

(b) $2x - 3 < 5x - 9$

	$2x - 3 < 5x - 9$
-5x)	$-3x - 3 < -9$
+3)	$-3x < -6$
÷-3)	$x > 2$



COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Equations > **Solving Absolute Value Equations**

Work through all Sub Lessons of the following Lessons **in order**:

- *Absolute Value… What is it?*
- *Absolute Value Equations in 1 Variable*
- *Absolute Value Inequalities in 1 Variable*
- *Absolute Value Equations in 2 Variable*

Additional Required Materials: Pencil Crayons

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

NOTES:

The absolute value measures the ***distance*** a number is away from the origin (***zero***) on the number line.

*Distance is always a **positive** number.*

*Absolute value is always a **positive** number.*

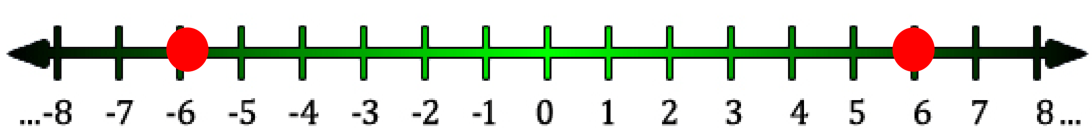
If you have | x | you can have two solutions for x:

If $x \geq 0$, then $|x| = x$

If $x < 0$, then $|x| = \mathbf{-x}$

Example:

What two numbers have an absolute value of 6? Show your answer on the number line.



Practice:

(a) $|x| = 4$

Then Case 1: $x \geq 0$ then $|x| = \mathbf{x}$

$\mathbf{x = 4}$

Case 2: $x < 0$ then $|x| = -(\mathbf{x})$

$\mathbf{-x = 4}$

$\mathbf{x = -4}$

(b) $|x + 4| = 8$

Then Case 1: $x+4 \geq 0$ then $|x + 4| = x + 4$

Case 2: $x+4 < 0$ then $|x + 4| = \mathbf{-(x + 4)}$

$\mathbf{= -x - 4}$

> Recall, if $(2x - 6) < 0$, then $|2x - 6| = -(2x - 6)$

Solving an equation with absolute value in it requires you to examine ***two*** cases.

Use the definition of absolute value to set up the two equations. The resulting linear equation is then solved. Finally you must ***check*** to see if the ***value of x*** makes the ***original equation*** true.

Practice:

$$| x + 4 | = 8$$

Case 1

$x+4 \geq 0$ then $| x + 4 | = \mathbf{x + 4}$

Rewrite the equation:

$$\boldsymbol{x + 4} = 8$$

Solve the linear equation:

$$\mathbf{x + 4} = 8$$

-4) $\quad x = \mathbf{4}$

Check: Substitute x = 4 into (1)

L.S. $| x + 4 | = | \mathbf{4} + 4|$

$= \mathbf{8}$

R. S = 8

Does it check?

Case 2

$x+4 < 0$ then $| x + 4 | = \mathbf{-(x + 4)}$

$= \mathbf{-x - 4}$

Rewrite the equation:

$$\mathbf{-x - 4} = 8$$

Solve the linear equation:

$$\mathbf{-x - 4} = 8$$

+4) $\quad \mathbf{-x} = \mathbf{12}$

×-1) $\quad x = \mathbf{-12}$

Check: Substitute x = 12 into (1)

L.S. $| x + 4 | = | \mathbf{-12} + 4|$

$= \mathbf{8}$

R. S = 8

Does it check?

OFF COMPUTER EXERCISES

1. The absolute value of a number is 5. What is the original number? **5 or -5**

2. Graph $| x | \leq 4$.

Case 1, x positive

$|x| \leq 4$

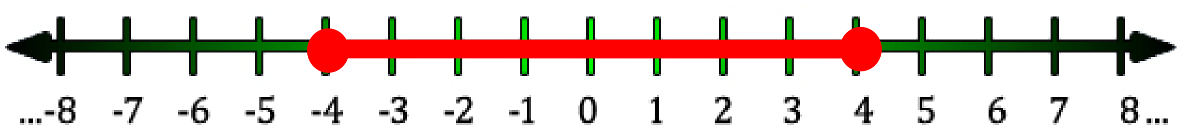
$\therefore \quad x \leq 4$

Case 1, x negative

$|x| \leq 4$

$-x \leq 4$

x-1) $\quad x \geq -4$



3. Solve

(a) $|2x - 6| = 10$

Case 1: $2x - 6 \geq 0$

then $|2x - 6| = 2x - 6$

Solve: $2x - 6 = 10$

$2x = 16$

$x = 8$

Check:

L.S. $= |2x - 6| = |2(8) - 6| = |10| = 10$ R.S. $= 10$

Case2: $2x - 6 < 0$

then $|2x - 6| = -(2x - 6) = -2x + 6$

Solve $-2x + 6 = 10$

-6) $-2x = 4$

÷-2) $x = -2$

Check:

L.S. $= |2x - 6| = |2(-2) - 6| = |-10| = 10$ R.S. $= 10$

(b) $|x + 4| = 1$

Case 1: $x + 4 \geq 0$

then $|x + 4| = x + 4$

Solve: $x + 4 = 1$

-x) $x = -3$

Check:

L.S. $= |x + 4| = |-3 + 4| = |1| = 1$ R.S. $= 1$

Case2: $x + 4 < 0$

then $|x + 4| = -(x + 4) = -x - 4$

Solve $-x - 4 = 1$

+4) $-x = 5$

×-1) $x = -5$

Check:

L.S. $= |x + 4| = |-5 + 4| = |-1| = 1$ R.S. $= 1$

(c) $|x - 2| = 4$

Case 1: $x - 2 \geq 0$

then $|x - 2| = x - 2$

Solve: $x - 2 = 4$

+2) $x = 6$

Check:

L.S. $= |x - 2|$ R.S. $= 4$

$= |6 - 2|$

$= |4|$

$= 4$

Case2: $x - 2 < 0$

then $|x - 2| = -(x - 2)$

$= -x + 2$

Solve $-x + 2 = 4$

-2) $-x = 2$

×-1) $x = -2$

Check:

L.S. $= |x - 2|$ R.S. $= 4$

$= |-2 - 2|$

$= |-4|$

$= 4$

(d) $|3x - 1| = 5$

Case 1: $3x - 1 \geq 0$

then $|3x - 1| = 3x - 1$

Solve: $3x - 1 = 5$

+1) $3x = 6$

÷3) $x = 2$

Check:

L.S. $= |3x - 1|$ R.S. $= 5$

$= |3(2) - 1|$

$= |5|$

$= 5$

Case2: $3x - 1 < 0$

then $|3x - 1| = -(3x - 1)$

$= -3x + 1$

Solve $-3x + 1 = 5$

-1) $-3x = 4$

÷-3) $x = \frac{-4}{3}$

Check:

L.S. $= |3x - 1|$ R.S. $= 5$

$= \left|3\left(\frac{-4}{3}\right) - 1\right|$

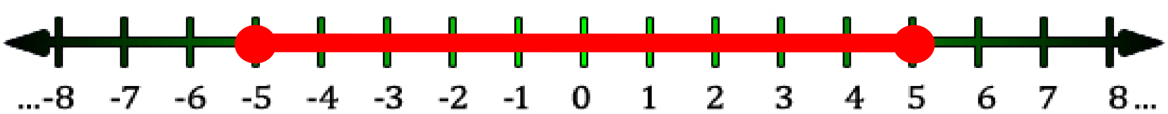
$= |-4 - 1|$

$= |-5|$

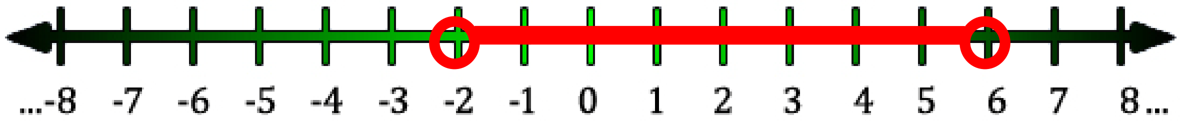
$= 5$

4. Graph the inequality

(a) $|x| \leq 5$



(b) $|x - 2| < 4$



5. Solve

(a) $|3 - x| \leq 8$

Case 1: $3 - x \geq 0$
then $|3 - x| = 3 - x$

Solve: $3 - x \leq 8$

-3) $-x \leq 5$
×-1) $x \geq -5$

Case2: $3 - x < 0$
then $|3 - x| = -(3 - x)$
$= -3 + x$

Solve $-3 + x \leq 8$

+3) $x \leq 11$

(N.B. Dividing or multiplying by a negative number reverses the sign)

(b) $|x - 2| < 2$

Case 1: $x - 2 \geq 0$
then $|x - 2| = x - 2$

Solve: $x - 2 < 2$

+2) $x < 4$

Case2: $x - 2 < 0$
then $|x - 2| = -(x - 2)$
$= -x + 2$

Solve $-x + 2 < 2$

-2) $-x < 0$
×-1) $x > 0$

N.B. Dividing or multiplying by a negative number reverses the sign)

(c) $|5x + 2| > 3$

Case 1: $5x + 2 \geq 0$
then $|5x + 2| = 5x + 2$

Solve: $5x + 2 > 3$
-2) $5x > 1$
÷5) $x > \frac{1}{5}$

Case2: $5x + 2 < 0$
then $|5x + 2| = -(5x + 2)$
$= -5x - 2$

Solve $-5x - 2 > 3$
+2) $-5x > 5$
÷-5) $x < -1$

N.B. Dividing or multiplying by a negative number reverses the sign)

(d) $|5x - 2| \le 3x + 1$

Case 1: $5x - 2 \ge 0$
then $|5x - 2| = 5x - 2$

Solve:
$5x - 2 \le 3x + 1$
+2) $5x \le 3x + 3$
-3x) $2x \le 3$
÷2) $x \le \frac{3}{2}$

Case2: $5x - 2 < 0$
then $|5x - 2| = -(5x - 2)$
$= -5x + 2$

Solve
$-5x + 2 \le 3x + 1$
-2) $-5x \le 3x - 1$
-3x) $-8x \le -1$
÷-8) $x \ge \frac{1}{8}$

N.B. Dividing or multiplying by a negative number reverses the sign)

6. Graph

(a) $y = |x + 2|$

Case 1: $x + 2 \ge 0$

then $|x + 2| = x + 2$
since $y = |x + 2|$
$y = x + 2$

y-intercept is (0, 2)
The slope of the line is 1

Isolate x to find which part of the line satisfies the condition

Using $x + 2 \ge 0$
$x + 2 - 2 \ge 0 - 2$
$x \ge -2$

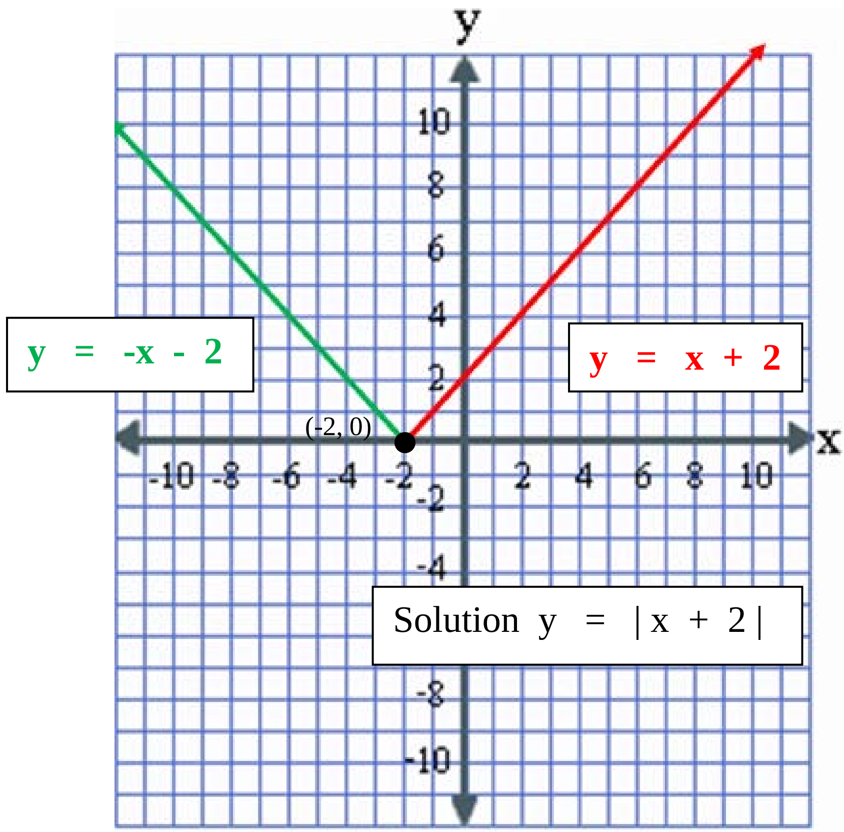
Case 2: $x + 2 < 0$

then $|x + 2| = -(x + 2)$
$= -x - 2$
since $y = |x + 2|$
$y = -x - 2$

y-intercept is (0,-2)
The slope of the line is -1

Isolate x to find which part of the line satisfies the condition

Using $x + 2 < 0$
$x + 2 - 2 < 0 - 2$
$x < -2$



(b) $y = 3|x| - 1$

Case 1: $x \geq 0$

then $|x| = x$
since $y = 3|x| - 1$
$y = 3x - 1$

y-intercept is (0,-1)
The slope of the line is 3

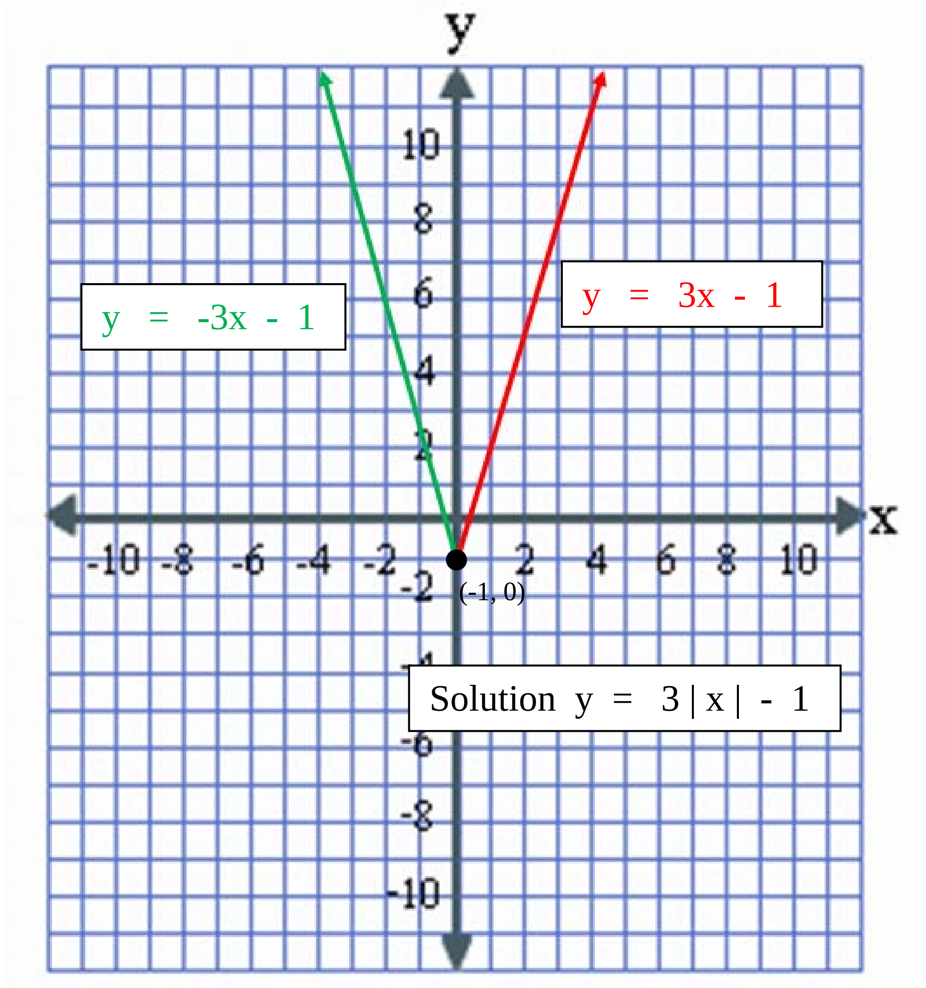
Graph $y = 3x - 1$ ***for*** $x \geq 0$

Case 2: $x < 0$

then $|x| = -x$
since $y = 3|x| - 1$
$y = -3x - 1$

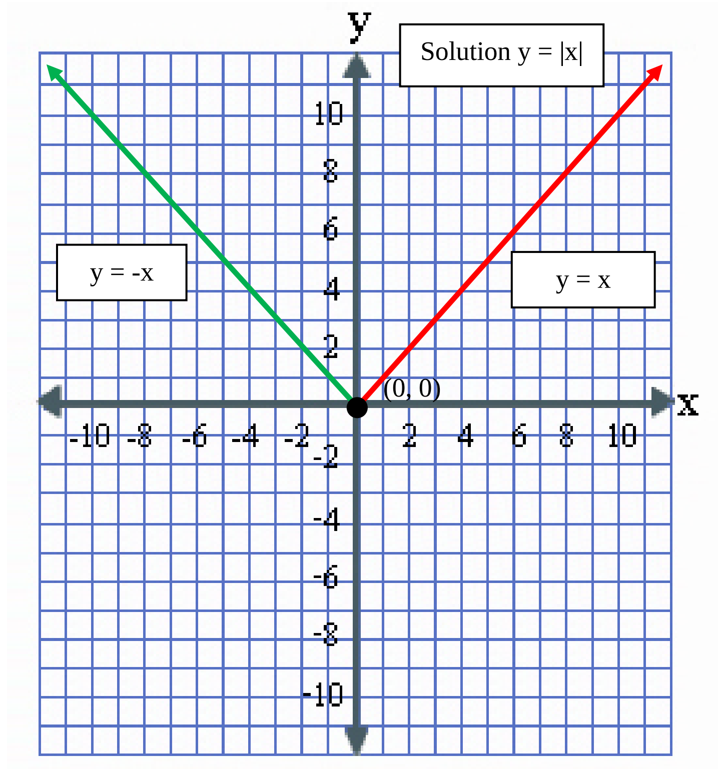
y-intercept is (0,-1)
The slope of the line is -3

Graph $y = -3x - 1$ ***for*** $x < 0$


y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 2 4 6 8 10
y = -3x - 1
y = 3x - 1
(-1, 0)
Solution y = 3 | x | - 1

7. Graph the following absolute value equation:

(a) y = | x |



Case 1 : For x ≥ 0

y = **x**

y-intercept (0 , **0**)

Slope of line **1**

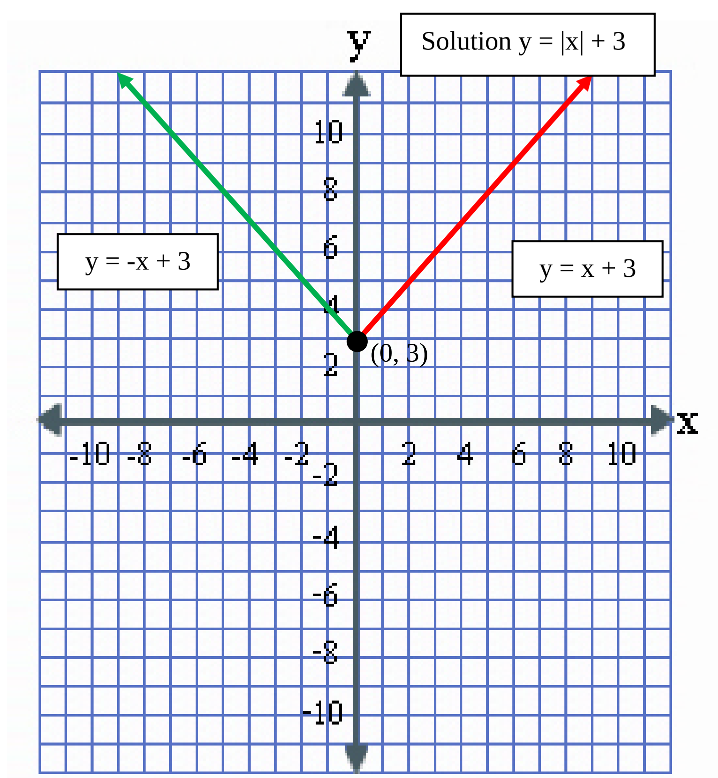
Case 2: For x < 0

y = **|x|**
= **-x**

y intercept (0 , **0**)

Slope of line **-1**

(b) y = | x | + 3



Case 1: For x ≥ 0

y = **x + 3**

y-intercept (0 , **3**)

Slope of line **1**

Case 2: For x < 0

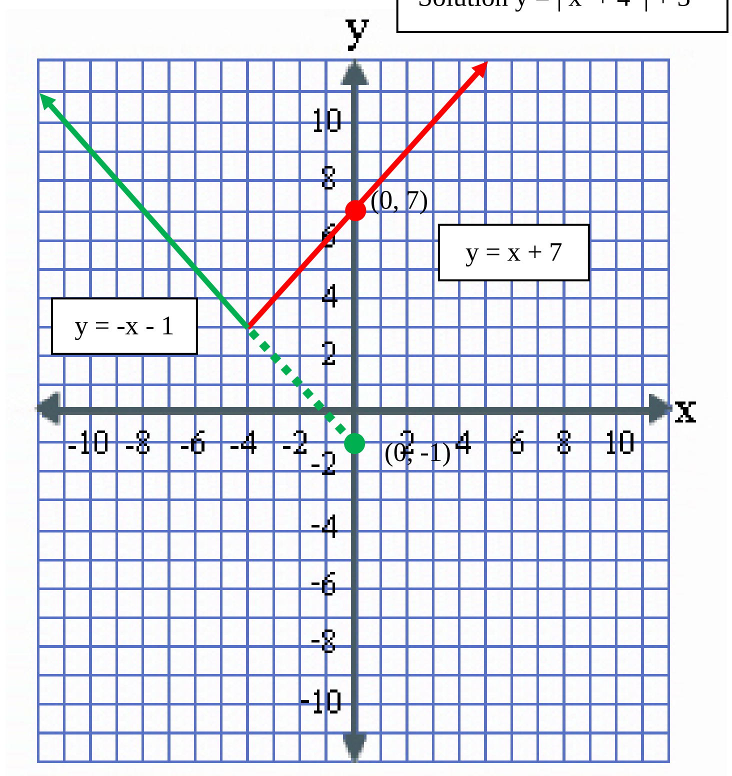
y = **-x + 3**

y intercept (0 , **3**)

Slope of line **-1**

(c) y = | x + 4 | + 3

Solution y = | x + 4 | + 3



Case 1: For x + 4 ≥ 0

y = **x + 4 + 3**
= **x + 7**

y-intercept (0 , **7**)
Slope of line **1**
Since x + 4 ≥ 0
x ≥ -4

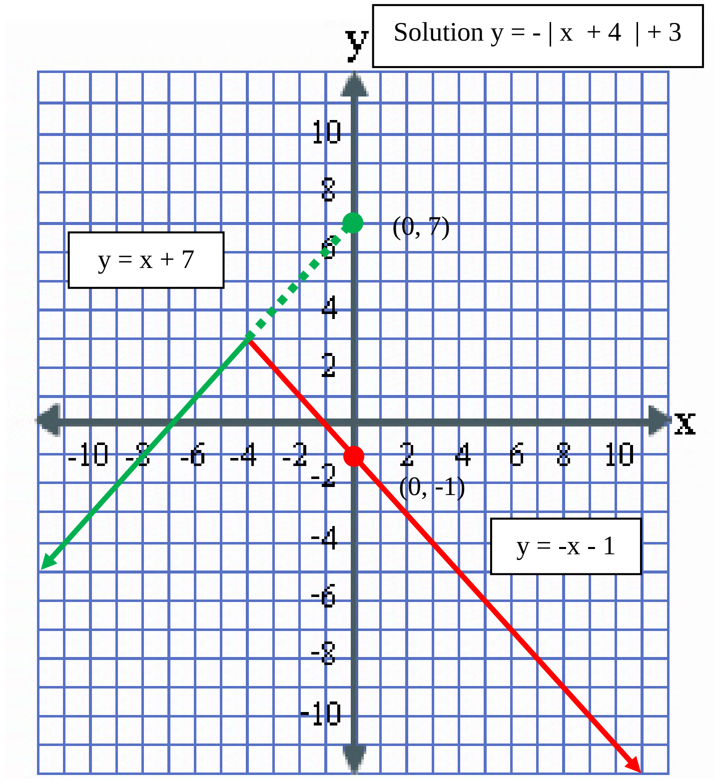
Case 2: For x + 4 < 0

y = **-x - 4 + 3**
= **-x - 1**

y intercept (0 , **-1**)
Slope of line **-1**
Since x + 4 < 0
x < -4

(d) y = - | x + 4 | + 3

Solution y = - | x + 4 | + 3



Case 1: For x + 4 ≥ 0

y = **-(x +4) + 3**
= **-x -1**

y-intercept (0 , **-1**)
Slope of line **-1**
Since x + 4 ≥ 0
x ≥ -4

Case 2: For x + 4 < 0

y = **-(-x -4) + 3**
= **x + 7**

y intercept (0 , **7**)
Slope of line **1**
Since x + 4 < 0
x < -4

(e) $y = -5|x+4|+3$

Case 1: For $x + 4 \geq 0$

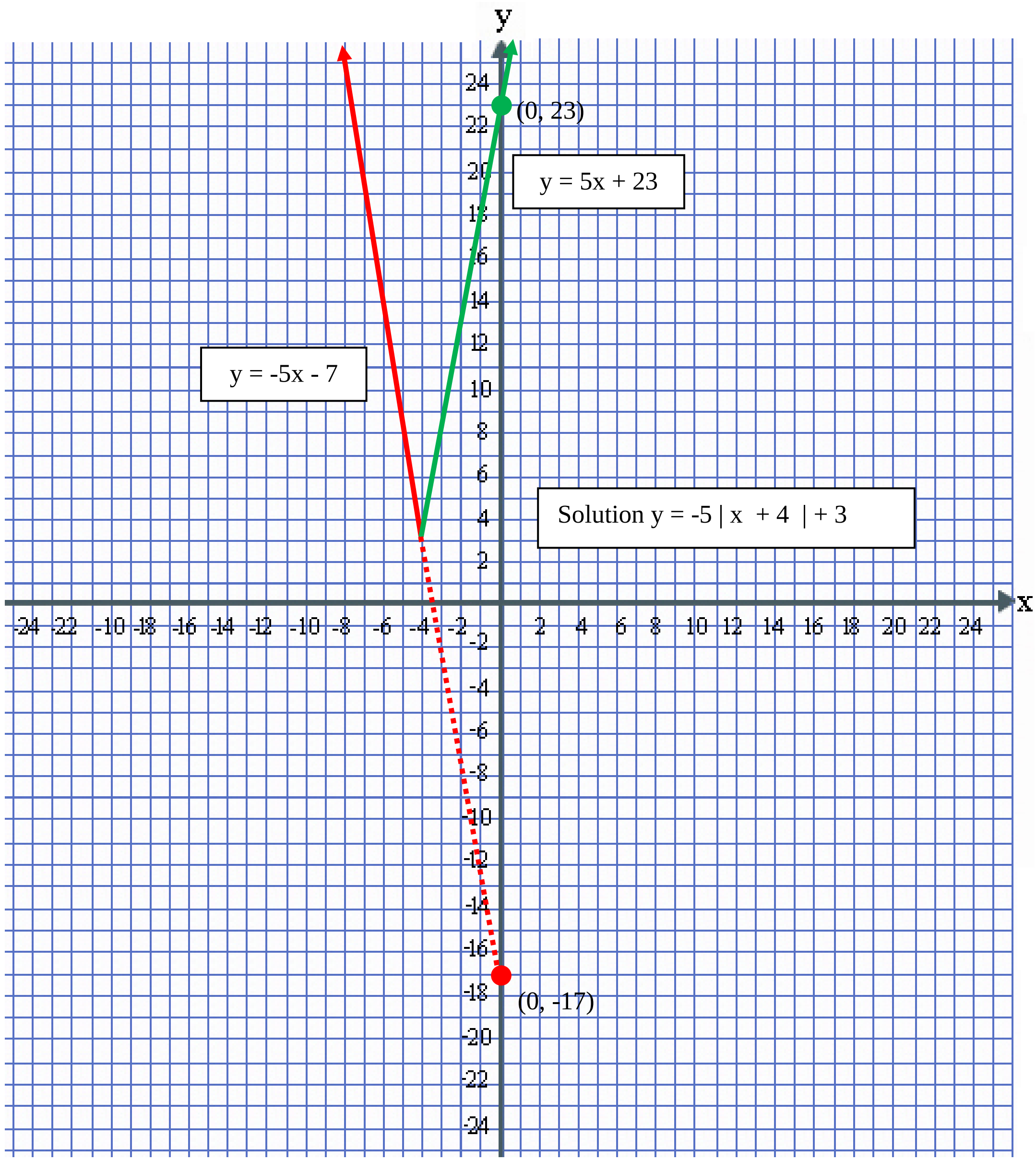
$y = \mathbf{-5(x+4)+3}$
$= \mathbf{-5x - 20 + 3}$
$= \mathbf{-5x - 17}$

y-intercept (0 , **-17**)
Slope of line **-5**
Since $x + 4 \geq 0$
$x \geq -4$

Case 2: For $x + 4 < 0$

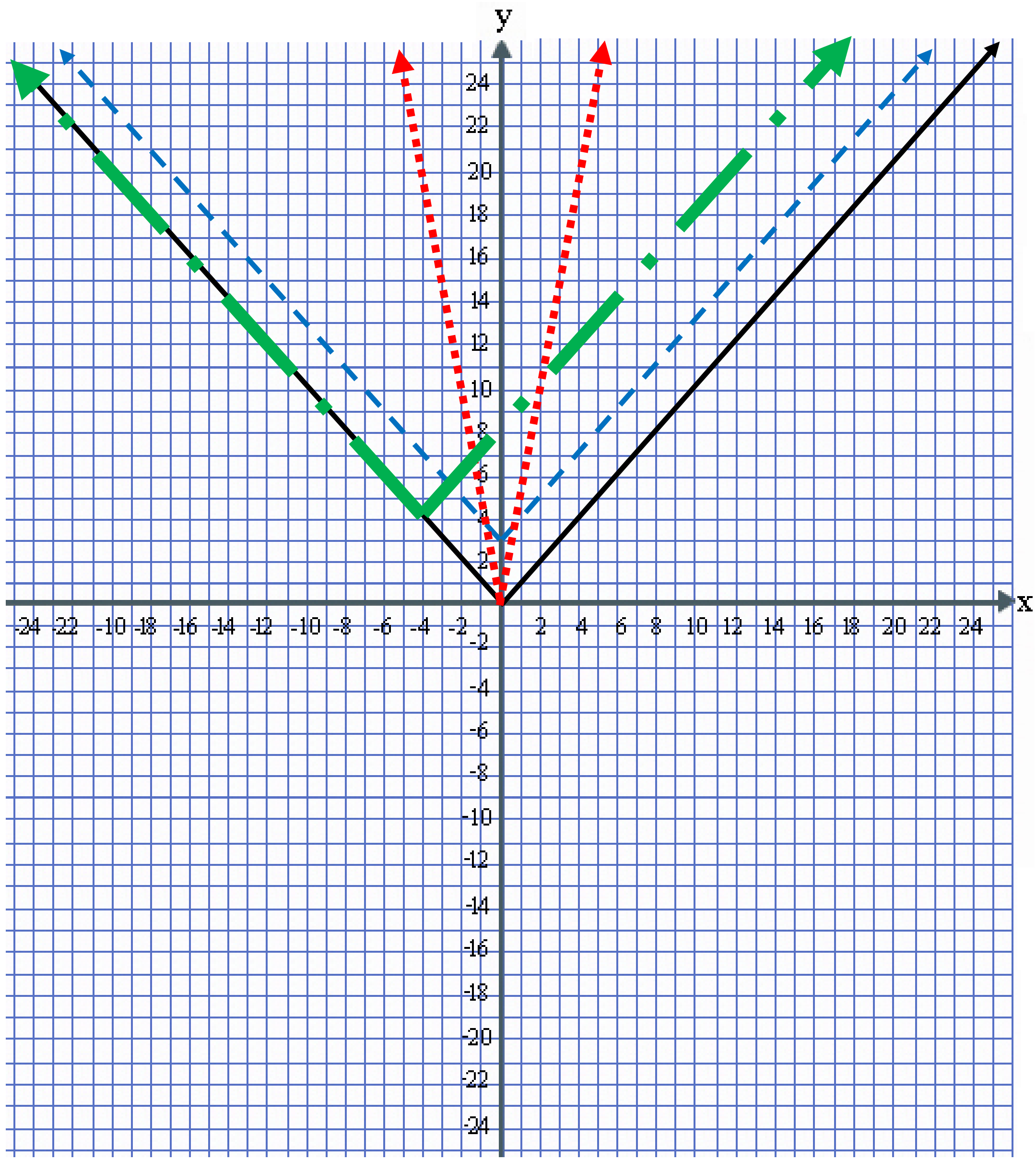
$y = \mathbf{-5(-x-4)+3}$
$= \mathbf{5x+2\text{-}+3}$
$= \mathbf{5x+23}$

y intercept (0 , **23**)
Slope of line is **5**
Since $x + 4 < 0$
$x < -4$



8. In the above graphs of the absolute value equations, + 3, + 4, "-"and - 5 were added to the equation y = | x |. *What effect did adding these numbers have on the graphs in relationship to their shape and position?*

Solution: graph y = |x| (done in black)



The -5 affected the slope of the lines, it made them steeper. y = |-5x| (red graph
The +4 shifted the point of intersection 4 units to the left. y = |x + 4| (green graph
The +3 raised the point of intersection 3 units up. y = |x| + 3 (blue graph