


Concept: Solving Linear Systems

Name:

- You should have completed Equations – Section 6 Part A: Solving Linear Systems before beginning this handout.

COMPUTER COMPONENT

Instructions: In  follow the **Content Menu** path:

Equations > Solving Linear Systems

NOTE: Use the **Menu** button in order to get to the lesson where you left off.



Work through all Sub Lessons of the following Lessons **in order**:

- *Solve a Linear System by Comparison*
- *Solve Problems Using Linear Systems*



As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

NOTES:

Solve a Linear System by Comparison (*Intersecting Lines*)

Step	Example
<p>1.</p> <p><u>Isolate</u> one of the <u>variables</u> for <u>each</u> equation. (We choose y.)</p>	$2x - y + 3 = 0$ $x - y - 1 = 0$ $y = 2x + 3 \quad (1)$ $y = 1x - 1 \quad (2)$
<p>2.</p> <p>For the <u>point of intersection</u> ,</p> <p><u>y</u> of (1) = <u>y</u> of (2)</p>	$(1) \qquad \qquad \qquad (2)$ $2x + 3 = 1x - 1$

<p>3.</p> <p>Solve for <u>one</u> variable.</p> <p>(<u>x</u> in this case)</p>	$2x + 3 = 1x - 1$ $2x - 1x = -1 - 3$ $1x = -4$ $x = -4$
<p>4.</p> <p>Substitute $x = \underline{-4}$ into one of the equations to solve for y.</p> <p>NOTE: <i>In this case substitute $x = -4$ into equation (1)</i></p>	$y = 2x + 3 \quad (1)$ $y = 2(-4) + 3$ $y = -5$ <p>Common point is (-4, -5)</p>

<p>5.</p> <p><u>Check</u> the solution in each <u>original</u> equation.</p> <p>NOTE: <i>In this case substitute $x = -4$ and $y = -5$ into equation (1)</i></p> <p>NOTE: <i>In this case substitute $x = -4$ and $y = -5$ into equation (2)</i></p>	$2x - y + 3 = 0 \quad (1)$ $x - y - 1 = 0 \quad (2)$ <p>For (1)</p> <p><i>Substitute $x = -4$ and $y = -5$ into equation (1)</i></p> $\begin{aligned} \text{L.S.} &= 2x - y + 3 \\ &= 2(-4) - (-5) + 3 \\ &= \underline{0} \end{aligned}$ <p>R.S. = 0 ← Same, L.S. = R.S.</p> <p>For (2)</p> <p><i>Substitute $x = -4$ and $y = -5$ into equation (2)</i></p> $\begin{aligned} \text{L.S.} &= x - y - 1 \\ &= (-4) - (-5) - 1 \\ &= \underline{0} \end{aligned}$ <p>R.S. = 0 ← Same, L.S. = R.S.</p> <p><i>Since L.S. = R.S. for both equations then the solution $x = -4, y = -5$ is correct.</i></p>
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Solving a Linear System by Comparison (Intersecting Lines Involving Fractions)

- For each equation, clear the fractions from the equation.

(multiply each term by a common denominator.)

Once we have cleared the fraction and expanded the bracket, you now have equations with which you can continue to solve using the above steps for solving a linear system by comparison (intersecting lines).

Solving a Linear System by Comparison (Parallel Lines)

- Parallel lines do not intersect. Therefore y of (1) cannot equal y of (2).

It is **not** possible to solve for **x** (or for **y**), and these types of

Linear Systems have **no solution**.

Solving a Linear System by Comparison (Coincidental Lines)

- Coincidental Lines are IDENTICAL. **All** points on line **one (1)** are also on line **two (2)**. This Linear System has an **infinite** number of **solutions**.

OFF COMPUTER EXERCISES

1. Solve the following linear systems by comparison.

$$\begin{aligned} \text{(a)} \quad y &= 2x + 3 \\ y &= x - 6 \end{aligned}$$

Isolate Variables

NOTE: *both equations are already expresses in terms of y.*

$$y = 2x + 3 \quad (1)$$

$$y = x - 6 \quad (2)$$

At the point of Intersection y in equation (1) = y in equation (2)

$$\begin{aligned} & 2x + 3 = x - 6 \\ -x) \quad 2x - x + 3 &= x - 6 - x \\ -3) \quad x + 3 - 3 &= -6 - 3 \\ & x = -9 \end{aligned}$$

Substitute x = -9 into equation (2)

$$\begin{aligned} y &= x - 6 \\ y &= -9 - 6 \\ y &= -15 \end{aligned} \quad (2)$$

The Solution Point is (-9, -15)

Check Equation (1) y = 2x + 3

$$\begin{array}{l} \text{L.S.} = y \\ \quad = -15 \end{array} \qquad \begin{array}{l} \text{R.S.} = 2x + 3 \\ \quad = 2(-9) + 3 \\ \quad = -18 + 3 \\ \quad = -15 \end{array}$$

Check Equation (2) y = x - 6

$$\begin{array}{l} \text{L.S.} = y \\ \quad = -15 \end{array} \qquad \begin{array}{l} \text{R.S.} = x - 6 \\ \quad = (-9) - 6 \\ \quad = -15 \end{array}$$

Since L.S. = R.S. for both equations, the solution x = -9, y = -15 is correct.

$$(b) \quad \begin{aligned} x + y - 4 &= 0 \\ 2x + y + 1 &= 0 \end{aligned}$$

Isolate Variables

NOTE: *both equations are already expresses in terms of y.*

$$y = -x + 4 \quad (1)$$

$$y = -2x - 1 \quad (2)$$

At the point of

Intersection y in equation (1) = y in equation (2)

+2x)

-4)

$$\begin{aligned} -x + 4 &= -2x - 1 \\ -x + 2x + 4 &= -2x + 2x - 1 \\ x + 4 - 4 &= -1 - 4 \\ x &= -5 \end{aligned}$$

Substitute $x = -5$ into equation (2)

$$y = -x + 4 \quad (2)$$

$$y = -(-5) + 4$$

$$y = 5 + 4$$

$$y = 9$$

The Solution Point is $(-5, 9)$

Check Equation (1) $x + y - 4 = 0$

$$\begin{aligned} L.S. &= x + y - 4 \\ &= -5 + 9 - 4 \\ &= 0 \end{aligned}$$

$$R.S. = 0$$

Check Equation (2) $2x + y + 1 = 0$

$$\begin{aligned} L.S. &= 2x + y + 1 \\ &= 2(-5) + 9 + 1 \\ &= 0 \end{aligned}$$

$$R.S. = 0$$

Since $L.S. = R.S.$ for both equations, the solution $x = -5, y = -9$ is correct.

$$\begin{aligned} \text{(c) } 5x + 2y - 8 &= 0 \\ 2x + 4y + 8 &= 0 \end{aligned}$$

Isolate Variables

Isolate y

$$y = \frac{-5x}{2} + 4 \quad (1)$$

At the point of

Intersection y in equation

(1) = y in equation (2)

$$y = \frac{-1x}{2} - 2 \quad (2)$$

Clear the fraction

$\times 2)$

$+x)$

$-8)$

$\div -4)$

$$\frac{-5x}{2} + 4 = \frac{-1x}{2} - 2$$

$$-5x + 8 = -x - 4$$

$$-4x + 8 = -4$$

$$-4x = -12$$

$$x = 3$$

Substitute $x = 3$ into equation (1)

$$y = \frac{-5x}{2} + 4 \quad (1)$$

$$y = \frac{-5}{2}(3) + 4$$

$$y = \frac{-15}{2} + \frac{8}{2}$$

$$y = \frac{-7}{2}$$

The Solution Point is $(3, \frac{-7}{2})$

Check Equation (1) $5x + 2y - 8 = 0$

$$\begin{aligned} \text{L.S.} &= 5x + 2y - 8 & \text{R.S.} &= 0 \\ &= 5(3) + 2(\frac{-7}{2}) - 8 \\ &= 15 - 7 - 8 \\ &= 0 \end{aligned}$$

Check Equation (2) $2x + 4y + 8 = 0$

$$\begin{aligned} \text{L.S.} &= 2x + 4y + 8 & \text{R.S.} &= 0 \\ &= 2(3) + 4(\frac{-7}{2}) + 8 \\ &= 6 - 14 + 8 \\ &= 0 \end{aligned}$$

Since L.S. = R.S. for both equations, the solution $x = 3, y = \frac{-7}{2}$ is correct.

$$(d) \quad 3x + 2y - 5 = 0 \quad (1)$$

$$4x + 3y - 2 = 0 \quad (2)$$

Isolate Variables

Isolate y

$$y = \frac{-3x}{2} + \frac{5}{2} \quad (3)$$

$$y = \frac{-4x}{3} + \frac{2}{3} \quad (4)$$

At the point of

Intersection y in equation

(3) = y in equation (4)

$$\frac{-3x}{2} + \frac{5}{2} = \frac{-4x}{3} + \frac{2}{3}$$

$\times 6)$

$$9x - 15 = 8x - 4$$

$-8x)$

$$x - 15 = -4$$

$+15)$

$$x = 11$$

Clear the fraction

*Substitute $x = 11$ into
equation (1)*

$$3x + 2y - 5 = 0 \quad (1)$$

$$3(11) + 2y - 5 = 0$$

$$33 + 2y - 5 = 0$$

$$2y + 28 = 0$$

$-28)$

$$2y = -28$$

$\div 2)$

$$y = -14$$

The Solution Point is (11, -14)

Check Equation (1) $3x + 2y - 5 = 0$

$$L.S. = 3x + 2y - 5$$

$$R.S. = 0$$

$$= 3(11) + 2(-14) - 5$$

$$= 33 - 28 - 5$$

$$= 0$$

Check Equation (2) $4x + 3y - 2 = 0$

$$L.S. = 4x + 3y - 2$$

$$R.S. = 0$$

$$= 4(11) + 3(-14) - 2$$

$$= 44 - 42 - 2$$

$$= 0$$

Since $L.S. = R.S.$ for both equations, the solution $x = 11, y = -14$ is correct.

2. The local fair charges \$6.00 for admission, plus \$0.50 for every ride ticket you buy. The neighboring town's fair offers free admission, but charges \$1.00 for every ride ticket. When is the local fair the better deal?

a) Create a table of values for each fair, let the number of rides be N and the cost be C .

Local Fair cost: $C = 6.00 + 0.50N$

N Number of Rides	1	2	3	4	5	6	7	8	9	10	11	12	13
C Cost (\$) of fair + rides	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50

Neighboring Town's Fair cost: $C = 1N$

N Number of Rides	1	2	3	4	5	6	7	8	9	10	11	12	13
C Cost (\$) of fair + rides	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00

b) Create the equation for each table:

$$C = 6.00 + 0.50N \quad (1)$$

$$C = 1N \quad (2)$$

c) Graph each line:

d) Point of Intersection
(12, 12.00)

e) Solve the two equations by comparison

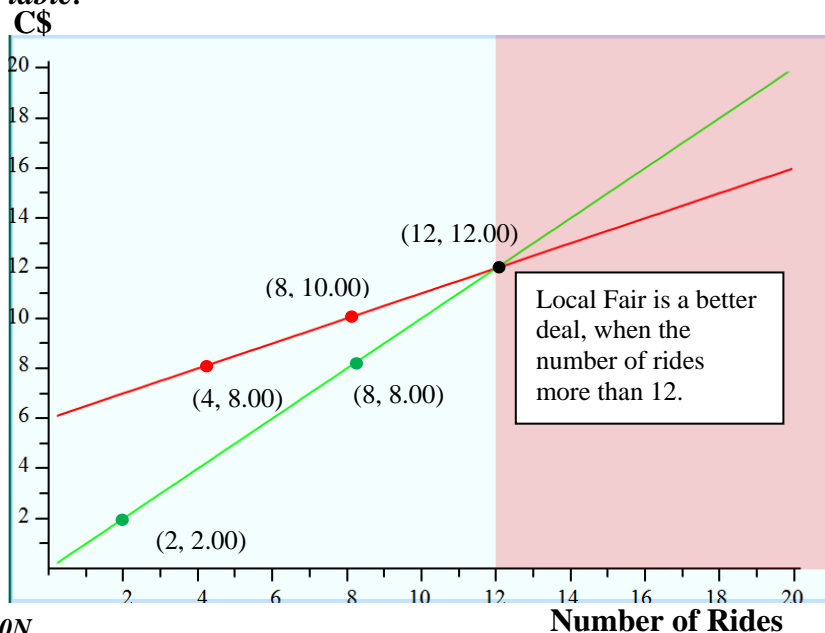
$$1N = 6.00 + 0.50N$$

$$1N - 0.50N = 6.00 + 0.50N - 0.50N$$

$$0.50N = 6.00$$

$\div 0.50$)

$$N = 12$$



f) Substitute $N = 12$ into equation (2)

$$C = 1N$$

$$C = 12.00$$

When is the local fair the better deal?

The local fair is better deal when the number of rides (N) is greater than 12.

3. The cost to rent a car with Company A is \$25 per day plus \$0.15 per km driven.
 The cost to rent a car with Company B is \$30 per day plus \$0.10 per km driven.
 Under what circumstances is Company A the better company to rent with?

a) Create a table of values for each company, let the number of km be N and the cost be C .

Company A cost: $C = 25 + 0.15N$

N Number of km	0	10	20	30	40	50	60	70	80	90	100	120
C Cost (\$) In dollars	25	26.50	28.00	29.50	31.00	32.50	34.00	35.50	37.00	38.50	40.00	41.50

Company B cost: $C = 30 + 0.10N$

N Number of km	0	10	20	30	40	50	60	70	80	90	100	120
C Cost (\$) In dollars	30	31.00	32.00	33.00	34.00	35.00	36.00	37.00	38.00	39.00	40.00	41.00

b) Create the equation for each table:

$$C = 25 + 0.15N \quad (1)$$

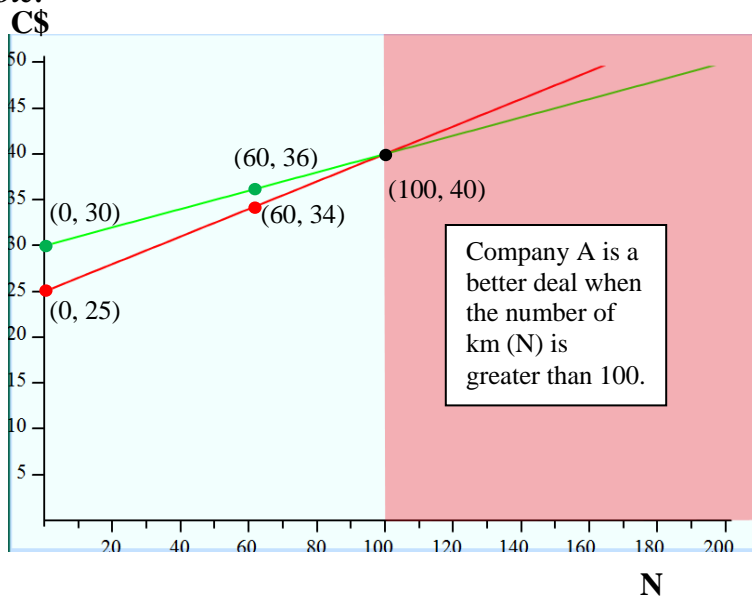
$$C = 30 + 0.10N \quad (2)$$

c) Graph each line:

d) Point of Intersection
(100, 40)

e) Solve the two equations by comparison

$$\begin{array}{rcl}
 25 + 0.15N & = & 30 + 0.10N \\
 -0.10N) \quad 25 + 0.05N & = & 30 \\
 -25) \quad 0.05N & = & 5 \\
 \div 0.05) \quad N & = & 100
 \end{array}$$



f) Substitute $N = 100$ into equation (2)

$$C = 30 + 0.10N$$

$$C = 30 + 0.10(100)$$

$$C = 30 + 40$$

$$C = 40$$

Under what circumstances is Company A the better company to rent with?

The company A is better deal when the number of km (N) is greater than 100.

4. The cost to rent a movie at Video Plus is \$2.00 for the first night plus \$0.50 for every night after that. The cost to rent a movie at Videos-R-Us is \$6.00 for 7 nights. *When is Videos-R-Us the better deal?*

a) *Create a table of values for each company, let the number of nights be N and the cost be C .*

Video Plus cost: $C = 2 + 0.50N$

N Number of Nights	0	7	14	21	28	35	42	49	56	63	70	77
C Cost (\$) In dollars	2	5.50	9.00	12.50	16.00	19.50	23.00	26.50	30.00	33.50	37.00	40.50

Videos-R-Us cost: $C = 6$ for 7 nights

N Number of Nights	0	7	14	21	28	35	42	49	56	63	70	77
C Cost (\$) In dollars	6	12	18	24	30	36	40	46	52	58	64	70

b) *Create the equation for each table:*

$$C = 2 + 0.50N \quad (1)$$

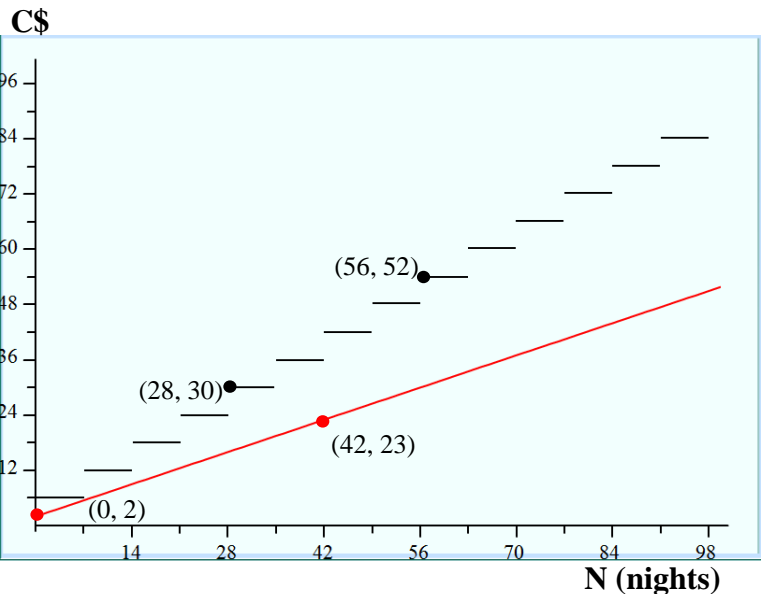
$$C = 6 \text{ for 7 nights} \quad (2)$$

c) *Graph each line:*

d) *Point of Intersection, there is no Point of Intersection.*

The graph shows that there is no point of intersection.

The Video Plus is always cheaper than Videos-R-Us.



*When is Videos-R-Us the better deal?
Video Plus is never a better deal.*

5. Which method would you choose to solve the given system of equations? Why?
 Justify your answer and then solve it.

$$6x + 4y = 23 \quad (1)$$

$$6x + 14y = 10 \quad (2)$$

The method I would choose is the Elimination Method. The x's are easily eliminated

Solution:

$$6x + 14y = 10 \quad (2)$$

$$6x + 4y = 23 \quad (1)$$

$$\begin{array}{r} (2) - (1) \\ \hline 0x + 10y = -13 \\ \div 10 \end{array} \quad \begin{array}{l} = \\ = \\ = \end{array} \quad \begin{array}{l} -13 \\ -1.3 \end{array}$$

Substitute $y = -1.3$ into (1)

$$6x + 4y = 23 \quad (1)$$

$$6x + 4(-1.3) = 23$$

$$6x - 5.2 = 23$$

$$+5.2) \quad 6x = 28.2$$

$$\div 6) \quad x = 4.7$$

The solution is (4.7, -1.3)

Check Equation (1) $6x + 4y = 23$

$$\begin{array}{l} L.R. = 6x + 4y \\ = 6(4.7) + 4(-1.3) \\ = 28.2 - 5.2 \\ = 23 \end{array} \quad \begin{array}{l} R.S = 23 \end{array}$$

Check Equation (2) $6x + 14y = 10$

$$\begin{array}{l} L.R. = 6x + 14y \\ = 6(4.7) + 14(-1.3) \\ = 28.2 - 18.2 \\ = 10 \end{array} \quad \begin{array}{l} R.S = 10 \end{array}$$

Since $L.S. = R.S.$ for both equations, the solution $x = 4.7, y = -1.3$ is correct.

6. A teacher hands out a math test to 36 students. The total marks for the test is 100 and it has 38 problems. The questions are worth either 5 marks or 2 marks. *How many questions of each type of mark are on the test? Justify your answer.*

NOTE: 36 students is an unnecessary piece of information.

Let x represent the number of 5 mark questions.

Let y represent the number of 2 mark questions.

$$\begin{aligned} \text{The total marks for the test} &= 100 \\ &= 5x + 2y \quad \therefore 5x + 2y = 100 \quad (1) \end{aligned}$$

$$\text{There are 38 problems} \quad \therefore x + y = 38 \quad (2)$$

Simplify and arrange terms

$$\begin{array}{rcl} & & 5x + 2y = 100 \quad (1) \\ & & x + y = 38 \quad (2) \\ & \left\{ \begin{array}{l} \text{Multiply equation (2) by } -1 \text{ so that} \\ \text{the } x \text{ terms add to 0.} \end{array} \right. & \begin{array}{r} (2) \times -1 \\ \hline 5x + 2y = 100 \quad (1) \\ -2x - 2y = -76 \quad (3) \end{array} \end{array}$$

$$\begin{array}{rcl} \text{Add equations (1) and (3)} & (1) + (3) & \begin{array}{r} 3x + 0y = 24 \\ 3x = 24 \\ \div 3 \\ x = 8 \end{array} \end{array}$$

$$\begin{array}{rcl} \text{Substitute } x = 8 \text{ into equation (2)} & & \\ \text{and solve for } y. & & \begin{array}{r} x + y = 38 \quad (2) \\ 8 + y = 38 \\ -8) \quad \quad y = 30 \end{array} \end{array}$$

There are 8 five mark questions and 30 two mark questions on the test.

Check equation (1) $5x + 2y = 100$

$$\begin{array}{lcl} \text{L. S.} & = & 5x + 2y \\ & = & 5(8) + 2(30) \\ & = & 40 + 60 \\ & = & 100 \end{array} \quad \text{R.S.} = 100$$

Check equation (2) $x + y = 38$

$$\begin{array}{lcl} \text{L. S.} & = & x + y \\ & = & 8 + 30 \\ & = & 38 \end{array} \quad \text{R.S.} = 38$$

Since L.S. = R.S. for both equations, the solution $x = 8, y = 30$ is correct.