Concept: Solving Linear Systems

Name:

COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Equations > **Solving Linear Systems**

Work through all Sub Lessons of the following Lessons **in order**:

- *In This Topic*
- *The Meaning of a Linear System*
- *Solve a Linear System by Graphing*
- *Solve a Linear System by Substitution*
- *Solve a Linear System by Elimination*

Additional Required Materials: *Pencil Crayons*

NOTE: You will not be finishing the entire section before stopping to complete some **OFF COMPUTER EXERCISES**.

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

When you reach the end of the lesson *Solve a Linear System by Elimination* on the computer, move on to the **OFF COMPUTER EXERCISES** below.

NOTES:

Remember from Understanding Graphing, Linear Relations, What is a Linear Relation?

- A ***line*** can be represented by a ***linear*** equation.
- A ***linear*** equation is of the form ***ax*** + ***by*** + ***c*** = ***0***

Where ***a***, ***b,*** and ***c*** represent ***real*** numbers.

- **y** = 3**x** + **2** is the equation of a ***straight*** line.
- For an equation of a straight line we can graph it by two methods:

Example: $y = 3x + 2$

- Method 1:

Pick ***many*** points on the ***line*** by picking some

values for ***x*** and finding the corresponding value for ***y***

Then:

Graph the points.

- Method 2:

  Determine

  - Slope (m) = **3**
  - y-intercept (b) = **2**
  - Then graph the y-intercept
  - Use the slope to find another point on the line with ***integer*** coordinates.

Summary:

3x + -y + 2 = 0 or y = 3x + 2 is a ***linear*** equation.

A ***pair*** of ***linear*** equations, considered **together,** is a ***Linear System.***

- When we ***solve*** a ***linear system*** we find the point of ***intersection.***

Solve a Linear System by Graphing

- We use the graphs of ***straight*** lines to ***estimate*** the point of ***intersection*** of the ***two*** lines.

Solve:

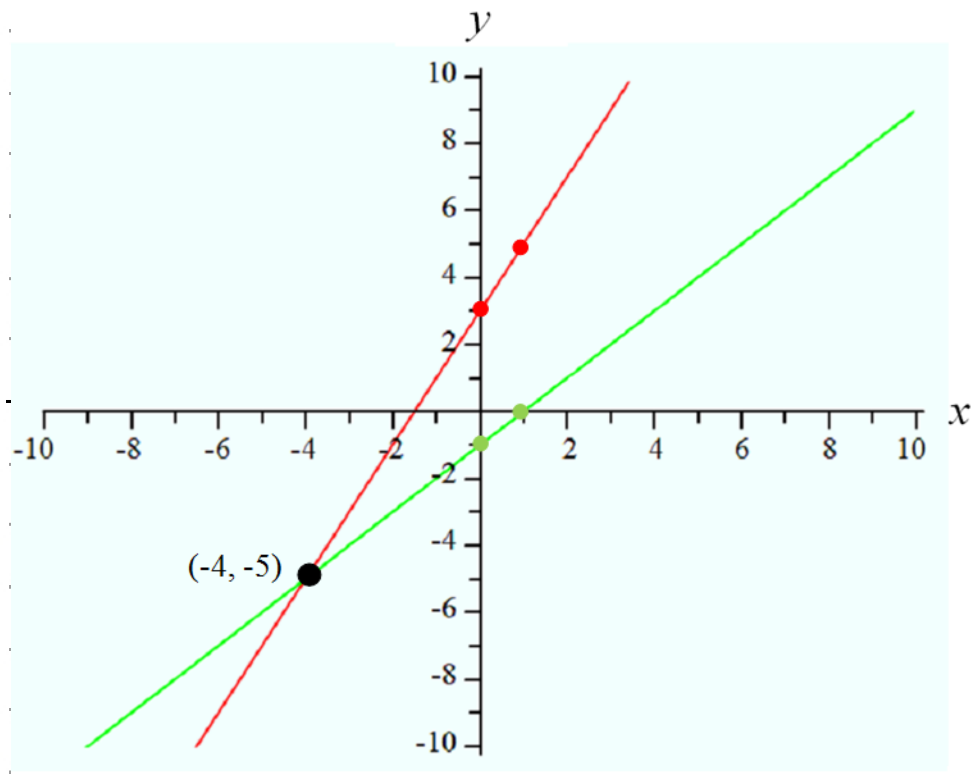
Equation	y-intercept	Slope	Another Integer Coordinate
y = 2x + 3	(0, **3**)	**2**	(**1** , **5**)
y = x - 1	(0, **-1**)	**1**	(**1** , **0**)

- Graph the coordinate points.
- Find the point where the ***2*** lines seem to ***intersect.***
- x = **-4**

  y = **-5**

  (**-4**, **-5**)

This point is called the ***solution*** of the system of linear equations.



After looking at
Solving a Linear Systems by Graphing:

1. Equations having slopes that are negative.
2. Equations having slopes that include fractions (*rise over run*).
3. Equations having slopes that are the same (*parallel lines*).
4. Equations having slopes that are the same (*coincidental lines*).

Determine similarities and differences and explain why. (*Use examples to help clarify your ideas.*)

(Answers will vary)

Consider the following, do not solve or graph.

a) $y = -2x + 5$

$y = -3x + 1$

i. ***The lines have negative slopes that are different***
ii. ***The lines slope down from left to right***
iii. ***The lines will intersect at one point (they have different slopes), therefore there is one solution.***

b) $y = \frac{1}{2}x + 1$

$y = \frac{1}{5}x - 2$

i. ***The lines have positive slopes that are different***
ii. ***The lines slope up from left to right***
iii. ***The lines will intersect at one point.***

c) $y = 7x + 3$

$y = 7x - 1$

i. ***The lines have the same slopes but different y intercepts, the lines are parallel and do not intersect. The system has no solution.***

d) $y = 3x + 4$

$2y = 6x + 8$

i. ***The lines are equivalent, they are coincidental, they are on top of each other***
ii. ***Every point on either line is a solution point, there are an infinite number of solutions.***

e) $y = 2x + 1$

$y = -3x - 1$

i. ***One line slopes up from left to right, the other slopes down from left to right***
ii. ***The lines intersect at one point***
iii. ***The system of equations has one solution.***

Solving a Linear System by Substitution

Solve:

$$2x - y + 3 = 0$$
$$x - y - 1 = 0$$

Steps:

Step	Example
1. Select ***one*** of the ***equations*** and ***isolate one of*** the ***variables***	$2x - y + 3 = 0$ (1) $x - y - 1 = 0$ (2) Choose equation (1) $y = \mathbf{2x + 3}$
2. ***Substitute*** this expression into the other ***equation*** (2)	$x - y - 1 = 0$ (2) $x - \mathbf{(2x + 3)} - 1 = 0$
3. Solve for the ***other*** variable. ***(In this case, solve for x)***	$x - \mathbf{(2x + 3)} - 1 = 0$ $\mathbf{x - 2x - 3 - 1 = 0}$ $\mathbf{-x - 4 = 0}$ **+x)** $\mathbf{-4 = x}$
4. ***Substitute*** this value (x = ***-4***) into the ***original*** equation to solve for ***y.***	$2x - y + 3 = 0$ $2\,\mathbf{(-4)} - y + 3 = 0$ $\mathbf{-8 - y + 3 = 0}$ **+y)** $\mathbf{-5 - y + y = 0 + y}$ $\mathbf{-5} = y$ Common point is (**-4**, **-5**)

5.
 	
Check***: Substitute*** the solution into each ***original*** equation.
	$2x - y + 3 = 0$ (1)
	$x - y - 1 = 0$ (2)
	
	For (1)
	
	L.S. $= 2x - y + 3$
	$= 2(\mathbf{-4}) - (\mathbf{-5}) + 3$
	$= \mathbf{0}$
	
	R.S. $= 0$
	
	Same, L.S. = R.S.
	
	For (2)
	
	L.S. $= x - y - 1$
	$= (\mathbf{-4}) - (\mathbf{-5}) - 1$
	$= \mathbf{0}$
	
	R.S. $= 0$
	
	Same, L.S. = R.S.

After looking at Solving a Linear Systems by Substitution:

1. Intersecting Lines.
2. Intersecting Lines Involving Fractions
3. Parallel Lines
4. Coincidental Lines.

Determine similarities and differences and explain why. (*Use examples to help clarify your ideas*.)

(Answers will vary)

Examine the following systems of equations, grouped according to similarities.

a) $3x - y + 2 = 0$
$y = x - 2$

c) $y = -2x - 1$
$y = x + 2$

b) $y = 3x + 5$

$4x + 3y = 32$

d) $y = \frac{1}{2}x + 4$

$y = \frac{-5}{2}x - 2$

a) and b)
In each of these systems of equations, y is isolated in one of the equations. Substitute this value for y into the other equation and solve for x. Then solve for y.

c) and d)
Substitute the value of y from one equation into the other equation, then solve for x. Clear fractions when necessary.

Solving Linear Systems by Elimination

Step	Example
1. ***Arrange*** the equations so that the **x**, **y**, and ***constant*** terms are ***above*** each other.	$2x - y + 3 = 0$ (1) $x - y - 1 = 0$ (2) $2x - y = \mathbf{-3}$ (1) $x - y = \mathbf{1}$ (2)
2. If ***necessary multiply*** each ***equation*** by a ***number*** so that the ***x*** or ***y*** terms ***add*** to ***0.***	(2) × - 1 → $2x - y = \mathbf{-3}$ $-x + y = \mathbf{-1}$
3. ***Add*** the equations to solve for ***one*** of the ***variables***	$2x - y = \mathbf{-3}$ $-x + y = \mathbf{-1}$ ――――――― $\mathbf{1}x + 0y = \mathbf{-4}$
4. ***Solve*** for (***x***).	$x = \mathbf{-4}$

5. ***Substitute*** for the ***variable*** into ***one*** equation and ***solve*** for the ***other variable***	$x - y - 1 = 0$ $(\mathbf{-4}) - y - 1 = 0$ $\mathbf{-5} = y$ Common point is (**-4, -5**) ***(Point of Intersection)***

6.
 	
Check the solution in each
 	
original equation.
	$2x - y + 3 = 0$ (1)
	$x - y - 1 = 0$ (2)
	
	The common point is (**-4**, **-5**)
	
	For (1) = 0
	
	L.S. = $x - y - 1$
	= $2(\mathbf{-4}) - (\mathbf{-5}) + 3$
	= **0**
	
	R.S. = 0
	
	Same, L.S. = R.S.
	
	For (2)
	
	L.S. = $x - y - 1$
	= $(\underline{\mathbf{-4}}) - (\underline{\mathbf{-5}}) - 1$
	= **0**
	
	R.S. = 0
	
	Same, L.S. = R.S.

Evaluate the methods used in Solving a Linear System by Elimination

1. Intersecting Lines.
2. Intersecting Lines Involving Fractions
3. Parallel Lines
4. Coincidental Lines.

Summaries the approaches used in each. Be as concise as you can.

Hints:

- *Remember the patterns that were outlined for solving linear systems using substitution and graphing for each along with the model for elimination that we just did.*
- *Use examples to support your ideas*.)

(Answers will vary)

a) Intersecting lines:

Systems of equations that intersect, whether or not involving fractions, can be solved by elimination of variable. When fractions are involved, every term in the equation is multiplied by a number that will clear the fraction.

See the following examples 1, 2, and 3

EXAMPLE 1

$$3x - y + 2 = 0$$
$$x - y - 2 = 0$$

Simplify and arrange terms

$$3x - y = -2 \quad (1)$$
$$x - y = 2 \quad (2)$$

Multiply equation (2) by -3 so that the x terms add to 0.

$$3x - y = -2 \quad (1)$$
(2) × -3 $$-3x + 3y = -6 \quad (3)$$

Add equations (1) and (3)

(1) + (3) $$0x + 2y = -8$$
÷2) $$y = -4$$

Substitute y = -4 into equation (2) and solve for x.

$$x - y - 2 = 0 \quad (2)$$
$$x + 4 - 2 = 0$$
$$x + 2 = 0$$
-2) $$x = -2$$

Common point (-2, -4)

The point of intersection is (-2,-4)
There is one solution (-2,-4)

EXAMPLE 2

$$2x + y + 1 = 0$$
$$x - y + 2 = 0$$

Simplify and arrange terms

$$2x + y = -1 \quad (1)$$
$$x - y = -2 \quad (2)$$

Solve using the elimination method
(NOTE: No change is needed because + y – y = 0)
Add equations (1) and (2)

(1) + (2): $3x + 0y = -3$

÷3): $x = -1$

Substitute x = -1 into equation (1) or (2)

Substitute x = -1 into equation (1) and solve for y

$$2x + y + 1 = 0 \quad (1)$$
$$2(-1) + y + 1 = 0$$
$$y = 1$$

Common point (-1, 1)

The point of intersection is (-1, 1)
There is one solution (-1, 1)

When the two equations describe lines that intersect at one point (x, y) the intersection point will satisfy the equation of either line. The point (x, y) is the one and only solution to the system of equations.

EXAMPLE 3 *(Equations involving fractions, elimination method)*

$$\frac{1}{3}x + \frac{1}{5}y = 2 \quad (1)$$
$$\frac{1}{3}x + \frac{1}{2}y = \frac{-1}{2} \quad (2)$$

Clear the fraction by multiply equation (1) by 15 and equation (2) by 6

×15): $15\left(\frac{1}{3}x + \frac{1}{5}y\right) = 15(2) \quad (1)$

$$5x + 3y = 30 \quad (3)$$

×6): $6\left(\frac{1}{3}x + \frac{1}{2}y\right) = 6\left(\frac{-1}{2}\right) \quad (2)$

$$2x + 3y = -3 \quad (4)$$

Multiply equation (4) by -1 so that the y terms add to 0.

Add equations (3) and (5)

$$
\begin{array}{rrcll}
 & 5x + 3y & = & 30 & (3) \\
(4)\ \times\text{-}1) & -2x - 3y & = & 3 & (5) \\
\hline
(3) + (5) & 3x + 0y & = & 33 & \\
\div 3) & x & = & 11 &
\end{array}
$$

Substitute x = 11 into equation (1), (2), (3) or (4)

Substitute x = 11 into equation (4) and solve for y

$$
\begin{array}{rcll}
2x + 3y & = & -3 & (4) \\
2(11) + 3y & = & -3 & \\
22 + 3y & = & -3 & \\
3y & = & -25 & \\
y & = & \frac{-25}{3} &
\end{array}
$$

Common point $\left(11, \frac{-25}{3}\right)$

The point of intersection is $\left(11, \frac{-25}{3}\right)$

There is one solution $\left(11, \frac{-25}{3}\right)$

b) Coincidental Lines

Systems of equations where each equation represents the same line (two lines on top of each other) do not have a unique solution. A solution attempt leads to a valid equation such as 3 = 3, or 7 = 7 or 0 = 0. This implies that every point on the line is a solution point.

See the following examples 4 and 5

EXAMPLE 4

$$
\begin{array}{rcll}
3x - 2y & = & 1 & (1) \\
4y & = & 6x - 2 & (2)
\end{array}
$$

Simplify and arrange terms

$$
\begin{array}{rrcll}
 & 3x - 2y & = & 1 & (1) \\
-6x) & -6x + 4y & = & -2 & (2)
\end{array}
$$

Multiply equation (1) by 2 so that the x terms add to 0.
NOTE: *Multiply equation (1) by 2 to get -6x which is opposite of 6x in equation (2).*

Add equation (3) and (2)

$$
\begin{array}{rrcll}
(1) \times 2 & 6x - 4y & = & 2 & (3) \\
 & -6x + 4y & = & -2 & (2) \\
\hline
(3) + (2) & 0x + 0y & = & 0 & \\
 & 0 & = & 0 & \text{(This is True)}
\end{array}
$$

There are an infinite number of points of intersection as both equations are equivalent.

EXAMPLE 5

$4x + 8y - 8 = 0$ (1)
$3x + 6y - 6 = 0$ (2)

Simplify and arrange terms

$4x + 8y = 8$ (1)
$3x + 6y = 6$ (2)

Multiply equation (1) by 3 so that the x terms becomes12x.
Multiply equation (2) by -4 so that the x terms becomes -12x.
NOTE*: The value of the x term in equation (1) is the opposite of the x term in equation (2) and the terms added (12x + -12x) is 0.*

(1) × 3 → $12x + 24y = 24$ (3)
(2) × -4 → $-12x - 24y = -24$ (4)

Add equation (3) and (4)

(3) + (4) $0x + 0y = 0$
$0 = 0$ (This is True)

There are an infinite number of points of intersection as both equations are equivalent. When the two equations describe lines that are equivalent, an algebraic solution for x and y does not exist. The two equations describe the same line, they have all their points in common and therefore there are an infinite number of solutions to the system.

c) Parallel Lines

Systems of equations that are parallel lines have no solution and an attempt to solve for a solution will lead to an equation that is false, such as 2 = 7. When a false equation results, this simply means that there is not a solution.

See the following examples 6 and 7

EXAMPLE 6

$2x - y - 3 = 0$
$2x - y + 1 = 0$

Simplify and arrange terms

$2x - y = 3$ (1)
$2x - y = -1$ (2)

Multiply the equations by a number so that x or y terms add to 0
NOTE*: Multiply equation (1) by -1 to get -2x* which *is opposite of 2x in equation (2).*

(1) × -1 → $-2x + y = -3$
$2x - y = -1$

Add equation (1) and (2)

(1) + (2) $0x + 0y = -4$
$0 = -4$ (Not Possible)

The slopes of these two lines are the same (-2) and they will never intersect. There is no point that could satisfy both equations as there is no point (x, y) that is on both lines. There is NO Solution.

EXAMPLE 7

$$3x + 2y - 6 = 0$$
$$3x + 2y - 12 = 0$$

Simplify and arrange terms

$$3x + 2y = 6 \quad (1)$$
$$3x + 2y = 12 \quad (2)$$

Multiply equations by a number so that either x or y terms add to 0

NOTE*: Multiply equation (2) by -1*

$$3x + 2y = 6$$
$$(2) \times -1 \quad -3x - 2y = -12$$

Add equations (1) and (2)

$$(1) + (2) \quad 0x + 0y = -6$$
$$0 = -6 \quad \text{(Not Possible)}$$

The slopes of these two lines are the same (3/2) and they will never intersect. There is no point that could satisfy both equations as there is no point (x, y) that is on both lines. There is NO Solution.

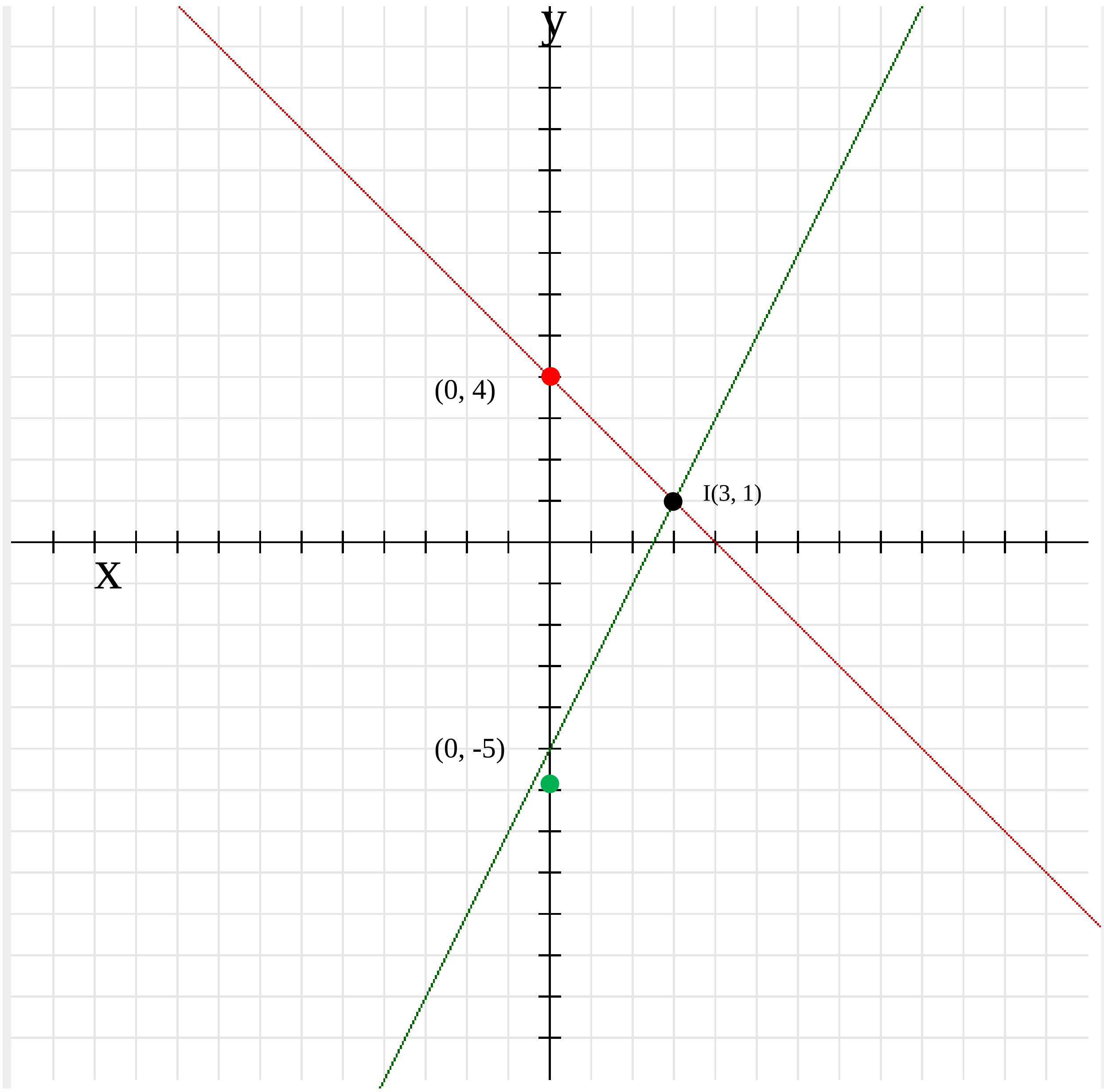
OFF COMPUTER EXERCISES

1. Find the solution to each system of linear equations by **graphing.**

(a) y = -x + 4 **(1)**
y = 2x - 5 **(2)**

y-intercept of line (1) is (0, 4)
Slope of line (1) is -1

y-intercept of line (2) is (0, -5)
Slope of line (2) is 2

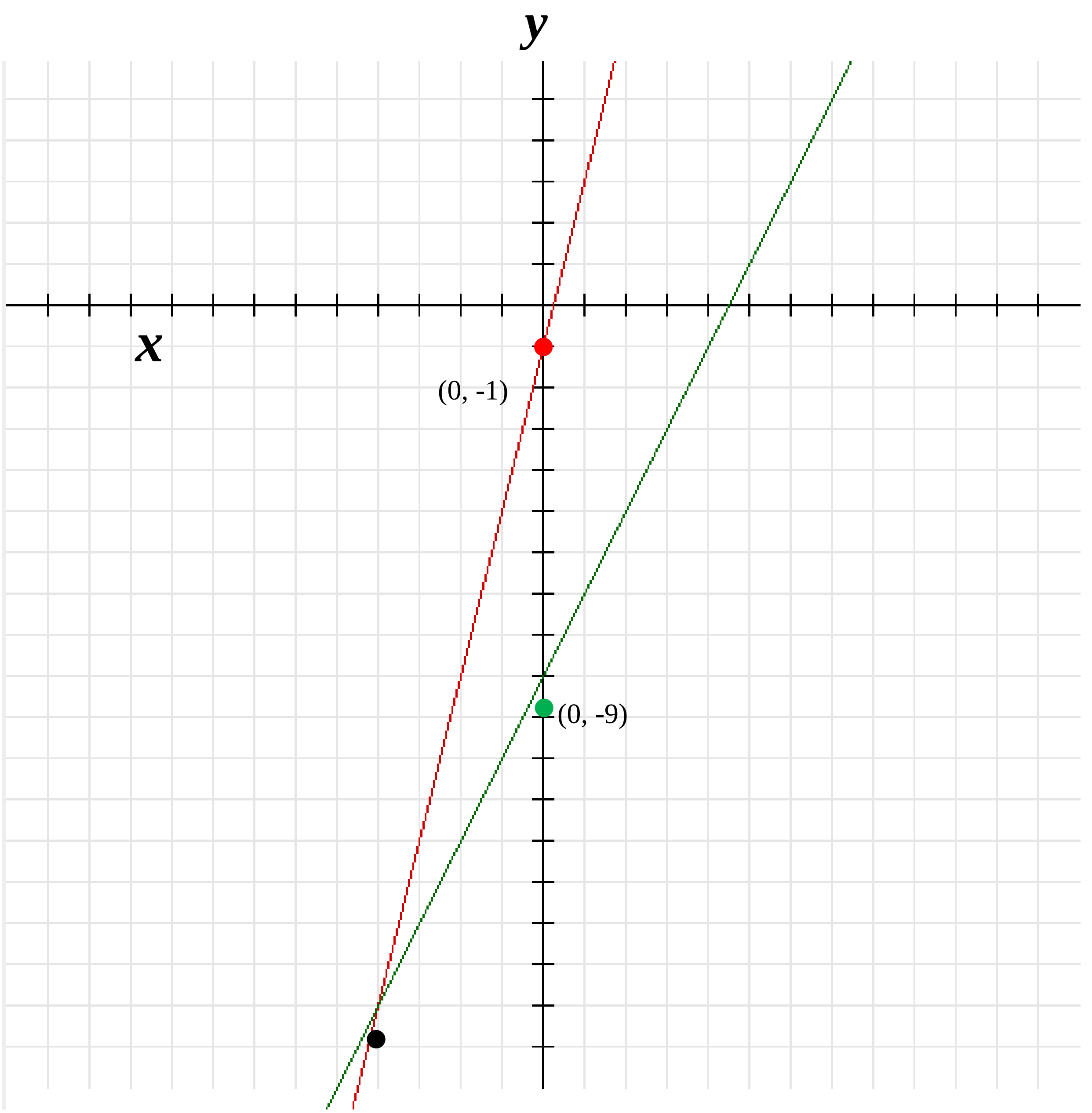


From the graph, the lines intersect at (3, 1) so the solution to the equations is x = 3, y = 1.

(b) y = 4x – 1 **(1)**
y = 2x – 9 **(2)**

y-intercept of line (1) is (0, -1)
Slope of line (1) is 4

y-intercept of line (2) is (0, -9)
Slope of line (2) is 2



From the graph, the lines intersect at (-4, -17) so the solution to the equations is x = -4, y = -17.

2. Solve the following linear systems by **substitution.** *(Show all your steps and make sure you check your solutions.)*

(a) $y = x + 1$ **(1)**

$2x + y + 5 = 0$ **(2)**

Select equation (1) $y = x + 1$

Substitute the value for y from equation (1) into equation (2)

$$\begin{aligned} 2x + y + 5 &= 0 \\ 2x + (x + 1) + 5 &= 0 \\ 2x + x + 1 + 5 &= 0 \\ 3x + 6 &= 0 \\ 3x + 6 - 6 &= 0 - 6 \\ 3x &= -6 \\ \frac{3x}{3} &= \frac{-6}{3} \\ x &= -2 \end{aligned}$$

Substitute x = -2 into equation (1) or (2)

Substitute x = -2 into equation (1)

$$\begin{aligned} y &= x + 1 \\ y &= (-2) + 1 \\ y &= -2 + 1 \\ y &= -1 \end{aligned}$$

The point of intersection (-2,-1)

Check equation (1) $y = x + 1$

$$\begin{aligned} L.S. &= y \\ &= -1 \end{aligned} \qquad \begin{aligned} R.S. &= x + 1 \\ &= -2 + 1 \\ &= -1 \end{aligned}$$

Check equation (2) $2x + y + 5 = 0$

$$\begin{aligned} L.S. &= 2x + y + 5 \\ &= 2(-2) + (-1) + 5 \\ &= -4 - 1 + 5 \\ &= 0 \end{aligned} \qquad R.S. = 0$$

Since L.S. = R.S. for both equations, the solution x = -2, y = -1 is correct.

(b) $x + y + 6 = 0$ **(1)**

$4x - y + 9 = 0$ **(2)**

Select equation (1) $x + y - 6 = 0$

Express equation (1) in terms of y:

	$x + y - 6$	=	0	(1)
-x)	$x + y + 6 - x$	=	$0 - x$	
-6)	$y + 6 - 6$	=	$-x - 6$	
	y	=	$-x - 6$	

Substitute y = -x -6 into equation (2)

	$4x - y + 9$	=	0	(2)
	$4x - (-x - 6) + 9$	=	0	
	$4x + x + 6 + 9$	=	0	
	$5x + 15$	=	0	
-15)	$5x + 15 - 15$	=	$0 - 15$	
	$5x$	=	-15	
	$\frac{5x}{5}$	=	$\frac{-15}{5}$	
	x	=	-3	

Substitute x = -3 into either equation (1) or equation (2)

Substitute x = -3 into (1)

$x + y + 6$	=	0	(1)
$-3 + y + 6$	=	0	
$y + 3$	=	0	
$y + 3 - 3$	=	$0 - 3$	
y	=	-3	

The point of intersection is (-3,-3)

Check equation (1) $x + y + 6 = 0$

L. S. $= x + y + 6$
$= -3 + (-3) + 6$
$= 0$

R.S. $= 0$

Check equation (2) $4x - y + 9 = 0$

L. S. $= 4x - y + 9$
$= 4(-3) - (-3) + 9$
$= -12 + 3 + 9$
$= 0$

R.S. $= 0$

Since L.S. = R.S. for both equations, the solution x = -3, y = -3 is correct.

(c) $5x + 2y = 14$ **(1)**

$8x + 4y - 28 = 0$ **(2)**

Select equation (2) $8x + 4y - 28 = 0$

Express equation (2) in terms of y:

	$8x + 4y - 28$	=	0	(2)
$-8x)$	$8x - 8x + 4y - 28$	=	$0 - 8x$	
$+28)$	$4y - 28 + 28$	=	$-8x + 28$	
	$4y$	=	$-8x + 28$	
$\div 4)$	$\frac{4y}{4}$	=	$\frac{-8x}{4} + \frac{28}{4}$	
	y	=	$-2x + 7$	

Substitute y = -2x + 7 into equation (1)

$5x + 2y$	=	14	(1)
$5x + 2(-2x + 7)$	=	14	
$5x - 4x + 14$	=	14	
$x + 14$	=	14	
$x + 14 - 14$	=	$14 - 14$	
x	=	0	

Substitute x = 0 into either equation (1) or equation (2)

Substitute x = 0 into equation (2)

$8x + 4y - 28$	=	0	(2)
$8(0) + 4y - 28$	=	0	
$4y - 28 + 28$	=	$0 + 28$	
$4y$	=	28	
$\frac{4y}{4}$	=	$\frac{28}{4}$	
y	=	7	

The point of intersection is (0, 7)

Check equation (1) $5x + 2y = 14$

L. S.	=	$5x + 2y$	**R.S.**	=	14
	=	$5(0) + 2(7)$			
	=	$0 + 14$			
	=	14			

Check equation (2) $8x + 4y - 28 = 0$

L. S.	=	$8x + 4y - 28$	**R.S.**	=	0
	=	$8(0) + 4(7) - 28$			
	=	$0 + 28 - 28$			
	=	0			

Since L.S. = R.S. for both equations, the solution x = 0, y = 7 is correct.

(d) $x - y = 10$ **(1)**

$2x - y = 16$ **(2)**

Select equation (1) $x - y = 10$

Express equation (1) in terms of y:

	$x - y$	=	10	**(1)**
$-x)$	$x - x - y$	=	$-x + 10$	
$\times -1)$	y	=	$x - 10$	

Substitute y = x – 10 into the equations (2)

	$2x - y$	=	16	**(2)**
	$2x - (x - 10)$	=	16	
	$2x - x + 10$	=	16	
	$x + 10$	=	16	
$-10)$	$x + 10 - 10$	=	$16 - 10$	
	x	=	6	

Substitute x = 6 into the equation (1)

	$x - y$	=	10	**(1)**
	$6 - y$	=	10	
$-6)$	$-y$	=	4	
$\times -1)$	y	=	-4	

The point of intersection is (6, -4)

Check equation (1) $x - y = 10$

L. S.	=	$x - y$	***R.S.***	=	10
	=	$6 - (-4)$			
	=	10			

Check equation (2) $2x - y = 16$

L. S.	=	$2x - y$	***R.S.***	=	16
	=	$2(6) - (-4))$			
	=	$12 + 4)$			
	=	16			

Since L.S. = R.S. for both equations, the solution x = 6, y = -4 is correct.

3. Solve the following linear systems by **elimination.** *(Show all your steps and make sure you check your solutions.)*

   (a) $x + 3y = 6$

   $2x - 3y = 12$

Simplify and arrange terms
Multiply the equations by a number so that either the x or y term add to 0
NOTE: No change is needed because +3y – 3y =0

$$x + 3y = 6 \quad (1)$$
$$2x - 3y = 12 \quad (2)$$

Add equation (1) and equation (2)
Solve for x:

(1) + (2) $$3x = 18$$

÷3) $$x = 6$$

Substitute x = 6 into equation (1) or equation (2)

Substitute x = 6 into equation (1)

$$x + 3y = 6 \quad (1)$$
$$6 + 3y = 6$$
$$6 - 6 + 3y = 6 - 6$$
$$3y = 0$$
÷3) $$y = 0$$

The point of intersection is (6,0)

Check equation (1) $x + 3y = 6$

L. S. $= x + 3y$ R.S. $= 6$
$= 6 + 3(0)$
$= 6$

Check equation (2) $2x - 3y = 12$

L. S. $= 2x - 3y$ R.S. $= 12$
$= 2(6) - 3(0)$
$= 12$

Since L.S. = R.S. for both equations, the solution x = 6, y = 0 is correct.

(b) $3x - 5y = 32$

$2x + y = 4$

Simplify and arrange terms

$$\begin{aligned} 3x - 5y &= 32 \quad (1) \\ 2x + y &= 4 \quad (2) \end{aligned}$$

Multiply the equations by a number so that either x or y terms add to 0

NOTE: *Multiply equation (2) by 5 to get 5y which is opposite of -5y in equation (1)*

(2) × 5

$$\begin{aligned} 5(2x + y) &= 4 \times 5 \quad (2) \\ 10x + 5y &= 20 \quad (3) \end{aligned}$$

Add equation (1) and (2)

$$\begin{aligned} 3x - 5y &= 32 \quad (1) \\ 10x + 5y &= 20 \quad (3) \\ \hline 13x + 0y &= 52 \\ x &= 4 \end{aligned}$$

Substitute x = 4 into equation (1) or equation (2)

Substitute x = 4 into equation (2)

$$\begin{aligned} 2x + y &= 4 \\ 2(4) + y &= 4 \\ 8 + y &= 4 \\ y &= -4 \end{aligned}$$

The point of intersection is (4,-4) -8)

Check equation (1) $3x - 5y = 32$

L. S. $= 3x - 5y$
$= 3(4) - 5(-4)$
$= 12 + 20$
$= 32$

R.S. $= 32$

Check equation (2) $2x + y = 4$

L. S. $= 2x + y$
$= 2(4) + (-4)$
$= 8 - 4$
4

R.S. $= 4$

Since L.S. = R.S. for both equations, the solution x = 4, y = -4 is correct.

(c) $3x + 2y = 5$
$4x + 3y = 2$

Simplify and arrange terms

$$3x + 2y = 5 \quad (1)$$
$$4x + 3y = 2 \quad (2)$$

Multiply the equations by a number so that either the x or y terms add to 0

NOTE: ***Multiply equation (1) by 3 to get 6y and multiply equation (2) by -2 to get -6y which is opposite of 6y in equation (3)***

(1) × 3
$$3(3x + 2y) = 5 \times 3$$
$$9x + 6y = 15 \quad (3)$$

(2) × -2
$$-2(4x + 3y) = 2 \times -2$$
$$-8x - 6y = -4 \quad (4)$$

$$9x + 6y = 15 \quad (3)$$
$$-8x - 6y = -4 \quad (4)$$

(3) + (4)
$$x = 11$$

Add equations (3) and (4)

Substitute x = 11 into equation (1) or equation (2)

$$3x + 2y = 5 \quad (1)$$
$$3(11) + 2y = 5$$
$$33 + 2y = 5$$
-33)
$$2y = -28$$
÷2)
$$y = -14$$

Substitute x = 11 into equation (1)

The point of intersection is (11,-14)

Check equation (1) 3x + 2y = 5

L. S. $= 3x + 2y$ **R.S.** $= 5$
$= 3(11) + 2(-14)$
$= 33 - 28$
$= 5$

Check equation (2) 4 x + 3 y = 2

L. S. $= 4x + 3y$ **R.S.** $= 2$
$= 4(11) + 3(-14)$
$= 44 - 42$
2

Since L.S. = R.S. for both equations, the solution x = 11, y = -14 is correct.

(d) $x + y = 11$
$-2x + 6y = 2$

Simplify and arrange terms		$x + y$	=	11	(1)
		$-2x + 6y$	=	2	(2)
Multiply the equations by a number so that either the x or y terms add to 0 **NOTE:** *Multiply equation (1) by 2 to get 2x which is opposite of -2x in equation (2)*	(1) × 2	$2(x + y)$	=	11 × 2	
		$2x + 2y$	=	22	(3)
Add equation (3) and equation (2)		$2x + 2y$	=	22	(3)
		$-2x + 6y$	=	2	(2)
Substitute y = 3 into either equation (1) or equation (2)	(3) + (2)	$8y$	=	24	(3)
	÷8)	y	=	3	(4)
Substitute y = 3 into equation (1)					
The point of intersection is (8, 3)		$x + y$	=	11	
		$x + 3$	=	11	
Check equation (1) $x + y = 11$	-3)	x	=	8	

L. S. $= x + y$ ***R.S.*** $= 11$
$= 8 + 3$
$= 11$

Check equation (2) $-2x + 6y = 2$

L. S. $= -2x + 6y$ ***R.S.*** $= 2$
$= -2(8) + 6(3)$
$= -16 + 18$
$= 2$

Since L.S. = R.S. for both equations, the solution x = 8, y = 3 is correct.