Concept: Problem Solving

Name:

- **You should have completed Equations – Section 5 Part A and B: Problem Solving before beginning this handout.**

COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Equations > **Problem Solving**

NOTE: *Use the **Menu** button in order to get to the lesson where you left off.*

Work through all Sub Lessons of the following Lessons **in order**:

- *Meat Mixture*
- *Coffee Mixture*
- *Rate of Work*
- *Summary – Problem Solving Using Equations*

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

NOTES:

Meat Mixture

Solve the following:

Mixers Supermarket combines meat that sells for $6.80/kg with meat that sells for $4.40/kg. They want a 30kg mixture that sells for $5.20/kg. *How many kilograms of each type of meat are used?*

Let **x** be the amount of ***$6.80*** /kg meat.

Then ***30 – x*** will be the amount of ***5.20*** /kg meat.

Fill in the table:

Meat	Amount	Value
6.80/kg	***x***	***6.80x***
4.40/kg	***30 – x***	***4.40(30 –x)***
5.20/kg	***30***	***5.20(30)***

We know the following: (*Write the equation and solve it.*)

The value of the meat at \$6.80/kg	plus	the value of the meat at \$4.40/kg	is equal to	the total value of the 30kg at \$5.10/kg
6.80x	+	***4.40(30 –x)***	=	***5.20(30)***

$$
\begin{aligned}
6.80x + 4.40(30 - x) &= 5.20(30) \\
6.80x + 132 - 4.40x &= 156 \\
2.4x + 132 &= 156 \\
2.4x + 132 - 132 &= 156 - 132 \quad (-132) \\
2.4x &= 24 \\
\frac{2.4x}{2.4} &= \frac{24}{2.4} \quad (\div 2.4) \\
x &= 10
\end{aligned}
$$

Check: Substitute x = 10 into (1)

$$
\begin{aligned}
\text{L.S.} &= 6.80x + 4.40(30 - x) \\
&= 6.8(10) + 4.40(30 - 10) \\
&= 68 + 4.40(20) \\
&= 68 + 88 \\
&= 156
\end{aligned}
\qquad
\begin{aligned}
\text{R.S.} &= 5.20(30) \\
&= 156
\end{aligned}
$$

L.S. = R.S. ***Therefore the Solution is correct.***

The amount of ***\$6.80***/kg meat is ***10*** kg.

The amount of ***\$4.40***/kg meat is $30 - x = 30 - 10$

$= 20$ *kg.*

Coffee Mixture

> Interesting Fact:
>
> Coffee is the second most traded commodity in the world. Oil is the first.
>
> http://www.gomestic.com/Consumer-Information/25-Facts-About-Coffee.42195

I have 25kg of coffee worth $7.00/kg. I want to add coffee worth $5.00/kg to create a blend worth $6.50/kg. How much should I add?
Record the steps required to answer the coffee mixture question.

Let y be the amount of $5.00/kg coffee
25 kg of coffee worth $7.00/kg is used.
Then 25 + y will be the amount of $6.50/kg coffee.

Fill in the table:

Coffee	Amount	Value
7.00/kg	*25*	*7.00(25)*
5.00/kg	*y*	*5.00y*
6.50/kg	*25 + y*	*6.50(25 + y)*

	$5.00y + 7.00(25)$	$=$	$6.50(25 + y)$	(1)
	$5.00y + 175$	$=$	$162.50 + 6.50y$	
$-6.50y)$	$5.00y + 175 - 6.50y$	$=$	$162.50 + 6.50y - 6.50y$	
	$-1.50y + 175$	$=$	162.50	
$- 175)$	$-1.50y + 175 - 175$	$=$	$162.50 - 175$	
	$-1.50y$	$=$	-12.50	
$\div -1.50)$	$\frac{-1.50y}{}$	$=$	$\frac{-12.50}{}$	
	-1.50	$=$	-1.50	
	y	$=$	8.3 (rounded to one decimal place)	

Check: Substitute y = 8.3 into (1)

L.S.	$=$	$5.00y + 7.00(25)$		***R.S.***	$=$	$6.50(25 + y)$
	$=$	$5.00(8.3) + 7.00(25)$			$=$	$6.50(25 + 8.3)$
	$=$	$41.5 + 175$			$=$	216.5
	$=$	216.5				

L.S. = R.S. Therefore the Solution is correct.

8.3 kg of the $5.00/kg coffee needs to be added.

Rate of Work

To solve rate of work problems:

(a) Determine what ***amount*** of the job gets done in ***one*** hour by each person.

Mary does $\frac{1}{2}$ of a car in an hour.

Juan does $\frac{2}{3}$ of a car in an hour

(b) Determine how much they can do in ***one*** hour if they work together.
(Hint: Add together what they can do individually.)

# of cars Mary does in one hour	+	# of cars Juan does in one hour	=	# of cars together in one hour
$\frac{1}{2}$	+	$\frac{2}{3}$	=	# of cars together in one hour
$\frac{1 \times 3}{2 \times 3}$	+	$\frac{2 \times 2}{3 \times 2}$	=	# of cars together in one hour
$\frac{3}{6}$	+	$\frac{4}{6}$	=	$\frac{7}{6}$

Hint: Common Denominator

(c) To determine how long it will take to was one car - divide one hour by how many cars they can wash in one hour.

Here is where our numbers start to make the problem look difficult. To understand what needs to be done – first use easier numbers.

If they wash 2 cars in an hour, to wash 1 car it takes $1 \div 2 = \frac{1}{2}$ hours.

If they wash 3 cars in an hour, to wash 1 car it takes $1 \div 3 = \frac{1}{3}$ hours.

The pattern:

If they wash $\frac{7}{6}$ of a car in an hour, to wash 1 car it takes $1 \div \frac{7}{6} = \frac{6}{7}$

Another way to look at the same problem.

Let the time taken for both of them to wash one car be t.

$\therefore$ $t \times$ # of cars Mary does in 1 hour $+ t \times$ # of cars Juan does in 1 hour $=$ 1 car

$$t \times \frac{1}{2} + t \times \frac{2}{3} = 1 \text{ car}$$

Hint: Common Denominator

$$t \times \frac{1 \times 3}{2 \times 3} + t \times \frac{2 \times 2}{3 \times 2} = 1 \text{ car}$$

$$t \times \frac{3}{6} + t \times \frac{4}{6} = 1 \text{ car}$$

$$\frac{7t}{6} = 1 \text{ car}$$

$$\frac{6}{7} \times \frac{7t}{6} = 1 \text{ car} \times \frac{6}{7}$$

Solve for t,

Hint: Multiply by the inverse.

$$t = \frac{6}{7} \text{ cars}$$

Working together they can was 1 car in $\frac{6}{7}$ of an hour.

Summary: Problem Solving Using Equations

Record the six steps in problem solving.

Step 1: ***Read*** the problem. ***Understand*** the problem.

Look for the ***unknown*** ***quantity***

Step 2: Let a ***variable*** represent an ***unknown*** quantity.

Step 3: ***Express*** any other ***unknown*** in terms of this ***variable.***

Make a ***diagram*** in ***area*** and ***perimeter*** questions.

Use a ***chart*** in money problems.

Use a ***chart*** in age problems.

Step 4: Write an ***Equation.*** Translate from ***words*** to ***math.***

Step 5: ***Solve*** the equation and clearly state the ***answer.***

Step 6: ***CHECK – Prove your answer is correct.***

OFF COMPUTER EXERCISES

Round all decimal answers to 1 decimal place.

1. A candy store combines gum drops that sell for $5.50/kg with gummy worms that sell for $4.00/kg. They want to end up with 81kg of mixture that will sell for $5.00/kg. *How many kilograms of each type of candy should they use?*

   Let x be the amount of $5.50/kg candy used.

   Then **81 - x** will be the amount of $4.00/kg candy used.

   Fill in the table:

	Amount	Value
gum drops ($5.50/kg)	x	***5.50(x)***
gummy worms ($4.00/kg)	***(81-x)***	***4.00(81 – x)***
mixture ($5.00/kg)	81	***5 × 81***

Solve for **x.**

Value of gum drops + Value of gummy worms = value of mixture

$$
\begin{aligned}
\therefore \quad & 5.5x + 4(81-x) = 5 \times 81 \\
& 5.5x - 4x + 324 = 405 \\
& 1.5x + 324 = 405 \\
-324) \quad & 1.5x + 324 - 324 = 405 - 324 \\
& 1.5x = 81 \\
& \frac{3x}{2} = 81 \\
& \frac{2}{} = \\
\times 2) \quad & (2)\frac{3x}{2} = (2)81 \\
& 3x = 162 \\
\div 3) \quad & \frac{3x}{3} = \frac{162}{3} \\
3 \quad & \\
& x = 54
\end{aligned}
$$

Check: Substitue x = 54 into 5.5x + 4(81-x) = 5 × 81

L.S.	=	***5.5x***	+	***4(81-x)***	***R.S.***	=	***81 × 5***
.	=	***5.5(54)***	+	***4(81-54)***		=	***405***
	=	***5.5(54)***	+	***4(81-54)***			
	=	***297***	+	***108***			
	=	***297***	+	***108***			
	=	***405***					

Number of kilograms of gum drops: = *x*
= ***54 kg***

Number of kilograms of gummy worms: = **(81-x)**
= **(81 – 54)**
= **27kg**

They should use ***54 kg*** of gum drops and ***27 kg*** of gummy worms.

2. A gardening store combines grass seed that sells for $2.65/kg with grass seed that sells for $3.25/kg. The store wants a 20kg mixture that will sell for $3.00/kg. *How many kilograms of each type of grass seed should they use*

Let **x** be the amount of $2.65/kg grass seed used.

Then ***20 – x*** will be the amount of $3.25/kg grass seed used.

Fill in the table:

	Amount	***Value***
Seed A	***x***	***2.65x***
Seed B	***20 - x***	***3.25(20 - x)***
Mixture ($3.00/kg)	***20***	***3.00 × 20***

Solve for x.

Value of Seed A + Value of Seed B = Value of Mixture

$\therefore$ ***2.65x + 3.25(20 - x) = 3.00 × 20***
2.65x + 3.25(20 - x) = 60

$$
\begin{aligned}
\times 100) \quad 265x + 325(20 - x) &= 6000 \\
265x + 6500 - 325x &= 6000 \\
-6500) \quad 265x + 6500 - 325x - 6500 &= 6000 - 6500 \\
265x - 325x &= -500 \\
-60x &= -500 \\
x &= 8.3 \text{ (rounded to one decimal place)}
\end{aligned}
$$

Check: Substitute $x = 8.3$ into $2.65x + 3.25(20-x) = 3.00 \times 20$

$$
\begin{aligned}
\text{L.S.} &= 2.65x + 3.25(20 - x) \\
&= 2.65(8.3) + 3.25(20 - 8.3) \\
&= 21.995 + 3.25(11.7) \\
&= 21.995 + 38.025 \\
&= 60.02 \\
&= 60 \text{ (rounded to 1 decimal place)}
\end{aligned}
\qquad
\begin{aligned}
\text{R.S.} &= 3.00 \times 20 \\
&= 60
\end{aligned}
$$

L.S. = R.S. Therefore the Solution is correct.

Number of kilograms of \$2.65 grass seed: $= x$
$= 8.3\text{kg}$

Number of kilograms of \$3.25 grass seed: $= 20 - x$
$= 20 - 8.3$
$= 11.7\text{kg}$

The store should use ***8.3 kg*** of Seed A and ***11.7 kg*** of Seed B.

3. Sally has 22kg of white flour worth \$1.25/kg. She wants to add whole wheat flour worth \$1.75/kg to create a blend worth \$1.40/kg. *How much whole wheat flour should she add?*

Flour Type	Amount	Value
Whole Wheat Flour	x	$1.75x$
White Flour	22	1.25×22
Blended Flour	$x + 22$	$1.40(x + 22)$

Let x be the amount of whole wheat flour.

Sally has ***22*** kg of white flour.

Then ***x + 22*** will be the amount of the blend.

Solve for x.

Value of WholeWheat + Value of White = Value of Blend

∴	$1.75x + 1.25 \times 22$	=	$1.40(x + 22)$	
$\times 100$)	$100(1.75x) + 100(1.25 \times 22)$	=	$100(1.40)(x + 22)$	
	$175x + 125 \times 22$	=	$140(x + 22)$	
	$175x + 2750$	=	$140x + 3080$	
$-140x$)	$35x + 2750$	=	3080	
-2750)	$35x$	=	330	
$\div 35$)	x	=	9.4	***(rounded to 1 decimal place)***

Check: Substitute x = 9.4 into 1.75x + 1.25 × 22 = 1.40(x + 22)

L.S.		***R.S.***	
=	$1.75x + 1.25 \times 22$	=	$1.40(x + 22)$
=	$1.75(9.4) + 1.25 \times 22$	=	$1.40(9.4 + 22)$
=	$16.45 + 27.5$	=	$1.40(31.4)$
=	43.95	=	43.96
=	44 ***(rounded)***	=	44 ***(rounded)***

L.S. = R.S. Therefore the Solution is correct.

Sally should add 9.4 kg of whole wheat flour.

4. Jeremy is able to pick an orchard of apples in 2 days when he works alone. Joshua is younger and is able to pick the same orchard of apples in 4 days when he works alone. They enjoy each other's company. How long would it take them if they worked together?

   Let x days be the time it takes to pick the orchard if they work together.

   Since Jeremy takes 2 days to pick the orchard, in 1 day he picks ***0.5*** of the orchard.

   Since Joshua takes 4 days to pick the orchard, in 1 day he picks ***0.25*** of the orchard.

   In x days, Jeremy can pick ***0.5x*** of the orchard

   Joshua can pick ***0.25x*** of the orchard.

   So when they work together to pick the orchard,

   Jeremy's work + Joshua's work = 1 **(*The whole job*)**

Continue on to finish the solution.

$$\therefore \quad 0.5x + 0.25x = 1$$
$$0.75x = 1$$
$$\text{or} \quad 3x = 1$$
$$x = 1$$

$$\times \frac{4}{3}) \quad \frac{4}{3} \times \frac{3x}{4} = 1 \times \frac{4}{3}$$
$$x = \frac{4}{3}$$

Check: ***Substitute*** $x = \frac{4}{3}$ ***into*** $0.5x + 0.25x = 1$

L.S. $= 0.5x + 0.25x$

$= (\frac{1}{2})(\frac{4}{3}) + (\frac{1}{4})(\frac{4}{3})$

$= \frac{4}{6} + \frac{1}{3}$

$\frac{4}{6} + \frac{2}{6}$

1

R.S. $= 1$

L.S. = R.S. Therefore the Solution is correct.

Working together they can pick all the apples in the orchard in 1 ⅓ days.

5. Bob has 1500g nails worth $4.45/kg. He wants to add nails worth $4.05/kg to create a blend worth $4.10/kg. *How many grams of the second type of nails should he add?*

(Hint: Use a chart in money problems)

Nails	*Amount*	*Value*
Type 1 nails	***1.5***	***4.45(1.5)***
Type 2 nails	***x***	***4.05x***
Blended Nails	***1.5 + x***	***4.10(1.5 + x)***

Let x represent the amount of added nails (kg).

Value of Type 1 + Value of Type 2 nails = Value of Blended Nails

∴	$4.45(1.5) + 4.05x$	=	$4.10(1.5 + x)$
×100)	$445(1.5) + 405x$	=	$410(1.5 + x)$
	$667.5 + 405x$	=	$615 + 410x$
-405x)	$667.5 + 405x - 405x$	=	$615 + 410x - 405x$
	667.5	=	$615 + 5x$
-615)	$667.5 - 615$	=	$615 + 5x - 615$
	52.5	=	$5x$
÷5)	$\frac{52.5}{5}$	=	$\frac{5x}{5}$
	10.5		x

Check: Substitute x = 10.5 into 4.45 (1.5) + 4.05x = 4.10(1.5 + x)

L.S.	=	$4.45(1.5) + 4.05x$	***R.S.*** = $4.10(1.5 + x)$
	=	$4.45(1.5) + 4.05(10.5)$	= $4.10(1.5 + 10.5)$
	=	$6.675 + 42.525$	= $6.15 + 43.05$
	=	49.2	= 49.2

L.S. = R.S. Therefore the Solution is correct.

Bob should use 10.5 kg or 1050 grams of the second type of nails.

6. If Joe works alone he can clean the house in 8 hours. Mary can clean the house alone in 6 hours. Joe and Mary decide to clean the house together. *How long will it take them?*

Name	*Hours/house*	*Houses/hour*
Joe	*8*	$\frac{1}{8}$
Mary	*6*	$\frac{1}{6}$

Working together,
Let the time it takes for them to clean 1 house be x hours.

∴	$\frac{1}{8}x + \frac{1}{6}x$	=	1
×24)	$3x + 4x$	=	24
	$7x$	=	24
÷7)	x	=	3.4 (rounded to one decimal places)

Check: Substitute x = 3.4 into $\frac{1}{8}x + \frac{1}{6}x = 1$

L.S. = $\frac{1}{8}x + \frac{1}{6}x$ ***R.S.*** = 1

$$= \frac{8}{8}1(3.4) + \frac{6}{6}1(3.4)$$
$$= \frac{3.4}{8} + \frac{3.4}{6}$$
$$= \frac{3.4}{8} \times \frac{3}{3} + \frac{3.4}{6} \times \frac{4}{4}$$
$$= \frac{10.2}{24} + \frac{13.6}{24}$$
$$= \frac{23.8}{24}$$
$$= 1 \text{ (rounded)}$$

L.S. = R.S. Therefore the Solution is correct.

It will take them 3.4 hours to clean the house when working together.

Another way:

In 8 hours Joe can clean 1 house.
In 6 hours Mary can clean 1 house

∴ *In 24 hours Joe can clean 3 houses.*
In 24 hours Mary can clean 4 houses.
Working together they can clean 7 houses in 24 hours
Working together they can clean 1 house in $\frac{24}{7}$ hours or 3.4 hours.

7. Juan can paint the house in two days when he works alone. Juanita can paint the house in three days when she works alone. *How long will it take Juan and Juanita to paint the house if they work together?*

Name	*Days/House*	*Houses/Day*
Juan	*2*	$\frac{1}{2}$
Juanita	*3*	$\frac{1}{3}$

When working together, let the time it takes for them to paint 1 house be x days.

∴ $\frac{1}{2}x + \frac{1}{3}x = 1$

×6) $3x + 2x = 6$

$5x = 6$

÷5) $x = 1.2$

Check: Substitute x = 1.2 into $\frac{1}{2}x + \frac{1}{3}x = 1$

$$L.S. = \frac{1}{2}x + \frac{1}{3}x \qquad R.S. = 1$$

$$= \frac{1}{2}(1.2) + \frac{1}{3}(1.2)$$

$$= \frac{1.2}{2} + \frac{1.2}{3}$$

$$= \frac{1.2}{2} \times \frac{3}{3} + \frac{1.2}{3} \times \frac{2}{2}$$

$$= \frac{3.6}{6} + \frac{2.4}{6}$$

$$= \frac{6}{6}$$

$$= 1$$

L.S. = R.S. Therefore the Solution is correct.

Working together, it will take Jaun and Juanita 1.2 days to paint the house.

8. It takes Max and Jennie two hours to mow the lawn when they work together. If Max works alone it takes him three hours to mow the lawn. *How long does it take Jennie to mow the lawn when she works alone?*

Name	***Hours/Lawn***	***Lawns/Hour***	***Lawns Mowed in 2 hours***
Max	***3***	$\frac{1}{3}$	$\frac{1}{3} \times 2$
Jennie	***x***	$\frac{1}{x}$	$\frac{1}{x} \times 2$

Let the time Jennie takes to mow the lawn to be represented by x.

Using the lawn mowed in 2 hours column and given that working together they mowed 1 lawn in 2 hours.

$$\therefore \quad \frac{1}{3} \times 2 + \frac{1}{x} \times 2 = 1$$

$$\times 3) \quad 2 + \frac{6}{x} = 3$$

$$-2) \quad \frac{6}{x} = 1$$

$$\times x) \quad 6 = x$$

Check: Substitute $x = 6$ into $\frac{1}{3} \times 2 + \frac{1}{x} \times 2 = 1$

$$\begin{aligned}
\textit{L.S.} &= \frac{1}{3} \times 2 + \frac{1}{x} \times 2 \qquad \textit{R.S.} = 1 \\
&= \frac{1}{3} \times 2 + \frac{1}{6} \times 2 \\
&= \frac{2}{3} + \frac{2}{6} \\
&= \frac{2}{3} \times \frac{2}{2} + \frac{2}{6} \\
&= \frac{4}{6} + \frac{2}{6} \\
&= \frac{6}{6} \\
&= 1
\end{aligned}$$

L.S. = R.S. Therefore the Solution is correct.

When working alone, Jennie will take 6 hours to mow the lawn.

Interesting Fact:

The Swiss lead the world in chocolate consumption per capita. They consume roughly 22 pounds of chocolate per person per year.

*www.geocities.com/NapaValley/4908/**trivia**.htm*

9. A chocolate factory wants to combine chocolates that sell for \$5.60/kg with chocolates that sell for \$6.20/kg. They want to end up with 24kg of mixture that will sell for \$6.00/kg. *How many grams of each type should they use?*

Type	*Amount*	*Value*
1st at 5.60/kg	*x*	*5.60x*
2nd at 6.20/kg	*y*	*6.20y*
Mixture 6.00/kg	*24*	*6.00(24)*

Let the amount of 5.60/kg chocolate be x kg.
Let the amount of 6.20/kg chocolate be y kg.

$$\begin{aligned}
\therefore \quad 5.6x + 6.20y &= 6.00 \times 24 \qquad (1) \\
x + y &= 24 \qquad (2)
\end{aligned}$$

Simplify and arrange terms

$$\therefore \quad 5.6x + 6.20y = 144 \qquad (1)$$

$x + y = 24$ (2)

Multiply equation (2)by -5.6 so that x terms add to 0

	$5.6x + 6.20y$	$=$	144	(1)
$\times -5.6)$	$-5.6x - 5.6y$	$=$	-134.4	(2)
(1) + (2)	$0x + 0.6y$	$=$	9.6	
	$0.6y$	$=$	9.6	
$\div 0.6)$	y	$=$	16	

Substitute y = 16 into the equations (2) x + y = 24
and solve for the other variable

	$x + y$	$=$	24 (2)
	$x + 16$	$=$	24
$-16)$	x	$=$	8

The amount of 5.60/kg chocolate is 8kg.
The amount of 6.20/kg chocolate is 16 kg.
Check: Substitute x = 8 and y = 16 into 5.6x + 6.20y = 6.00 × 24 (1)

L.S.	=	$5.6x + 6.20y$	R.S.	=	6.00×24
	=	$5.6(8) + 6.20(16)$		=	144
	=	$44.8 + 99.2$			
	=	144			

L.S. = R.S. Therefore the Solution is correct.

10. Uncle Al requires a large water tank to be filled in order to plant his tomato seeds.

He has three options for filling the water tank.

Option 1: He can use a large hose ... it requires 3 minutes to fill the tank.
Option 2: He can use a small hose ... it requires 5 minutes to fill the tank.
Option 3: He can use both hoses at the same time.

How long will it take to fill the tank using Option 3?

Option Number	Minutes/tank	Tank/minutes	Amount Filled in x minutes
Option 1	3	$\frac{1}{3}$	$\frac{1}{3} x$
Option 2	5	$\frac{1}{5}$	$\frac{1}{5} x$
Option 3	x		1

Let the time to fill 1 tank with option 3 be represented by x

$$\therefore \quad \frac{1}{3}x + \frac{1}{5}x = 1$$

$$\times 15) \quad 5x + 3x = 15$$

$$8x = 15$$

$$\div 8) \quad x = \frac{15}{8}$$

$$x = 1.875$$

Check: Substitute $x = \frac{15}{8}$ ***into*** $\frac{1}{3}x + \frac{1}{5}x = 1$

$$\text{L.S.} = \frac{1}{3}x + \frac{1}{5}x \qquad \text{R.S.} = 1$$

$$= \frac{1}{3} \times \frac{15}{8} + \frac{1}{5} \times \frac{15}{8}$$

$$= \frac{5}{8} + \frac{3}{8}$$

$$= \frac{8}{8}$$

$$= 1$$

L.S. = R.S. Therefore the Solution is correct.

Using both hoses (Option 3), he can fill the tank in 1.875 minutes.