Concept: Problem Solving

Name:

- **You should have completed Equations – Section 5 Part A: Problem Solving before beginning this handout.**

Warm Up

Translate the following word statements into symbols.

(a) ten times a number, increased by four ***10x + 4.***

(b) six less than twice a number ***2y - 6***

COMPUTER COMPONENT

Instructions: In UMATH X follow the **Content Menu** path:

Equations > **Problem Solving**

NOTE: *Use the **Menu** button in order to get to the lesson where you left off.*

Work through all Sub Lessons of the following Lessons **in order**:

- *Money Problem with Chart*
- *Age Problem with Chart*
- *Buying CDs*

NOTE*:* You will not be finishing the entire section before stopping to complete some **OFF COMPUTER EXERCES**.

As you work through the computer exercises, you will be prompted to make notes in your notebook/math journal.

When you reach the end of the lesson *Buying CDs* on the computer, move on to the **OFF COMPUTER EXERCISES** below**.**

NOTES:

Money Problem with Chart

A drawer contains $3.05 in quarters and dimes.
The number of dimes is 11 more than 4 times the number of quarters.

How many dimes and quarters are in the drawer?

	Dimes	Quarters
Number of	***4q + 11***	***q***
Value of (¢)	***10(4q +11)***	***25q***

The number of dimes is:

Number of quarters → **q**

4 times the number of quarters → **q**

11 more than **4** times the number of quarters → **4q + 11**

Value of the dimes:

10 (**4q + 11**)

Value of the quarters

25 (**q**)

Total value of the coins (*Hint: Remember to change the dollars into cents to keep units consistent*)

N.B. $3.05 = 305¢

$$10\ (4q + 11) + 25\ (q) = 305$$

$$40q + 110 + 25q = 305$$

$$65q + 110 = 305$$

$$65q = 305 - 110$$

$$65q = 195$$

$$q = 3$$

If q = **3** then…

Number of Quarters is q = **3**

Number of Dimes is **4q + 11** = 4 (**3**) + 11

= **23**

CHECK:

The value of the quarters (**25q**) = 25 (**3**)

= **75**

The value of the dimes (**10(4q+11**)) = 10 (**23**)

= **230**

Total value (¢) **75 + 230 = 305**

Age Problem with Chart

Mary is 8 times the age of her daughter
In 4 years she will be 4 times the age of her daughter plus 4 years

Fill in the chart:

	Present Age	**Age in 4 years**
Daughter	**y**	***y + 4***
Mary	***8y***	***4(y + 4) + 4 or 8y + 4***

There are two ways to write how old Mary will be in 4 years. *Write the expression for each.*

Current age plus number of years = Four times (daughter age in 4 years) plus 4 years

$$8y + 4 = 4(y + 4) + 4$$

Solve for y

$$\therefore \quad 8y + 4 = 4(y + 4) + 4$$
$$8y + 8 = 4y + 16 + 4$$

	$8y + 8$	=	$4y + 20$
$-4y$	$8y - 4y + 8$	=	$4y - 4y + 20$
	$4y + 8$	=	20
-8	$4y + 8 - 8$	=	$20 - 8$
	$4y$	=	12
$\div 4$	$\frac{4y}{3}$	=	$\frac{12}{3}$
	y	=	4

The daughter is presently **4** years old.

Mary is presently **8** y = **8 (4)**
= **32** years old.

CHECK:

Calculate her daughter's age in 4 years: **y + 4** = **(4) + 4**
= **8**

In four years Mary will be 4 times her daughters age plus 4 years = 4(**y** + ***4)*** + ***4***

= ***4(4 + 4)*** + ***4***

= ***36***

Mary's age in four years = 8 (**y**) + 4

= ***8(4)*** + 4

= ***36*** ← Does it match? ***Yes***

Buying CDs

The twins went shopping and spent $75.56 on 4 CDs and lunch.
They spent $12.56 for lunch.
The store had a sale on CDs, any CD for the same low price. The store also advertized that they were not charging tax that day! *What is the cost of each CD?*

Write the equation for calculating the cost of the CDs.

Let the cost of the CD be p
They spent $12.56 for lunch.
They spent 4p on CD's

Solve to find the cost of each CD.

$$
\begin{array}{lrcl}
 & 12.56 + 4p & = & 75.56 \\
-12.56) & 12.56 - 12.56 + 4p & = & 75.56 - 12.56 \\
 & 4p & = & 63.00 \\
\div 4) & \frac{4p}{4} & = & \frac{63.00}{4} \\
 & p & = & 15.75
\end{array}
$$

Therefore the cost of each CD is $15.75.

CHECK: $4 \times 15.75 + 12.56 = 75.56$ (The total amount that they spent)

OFF COMPUTER EXERCISES

1. Joe has $1.85 in dimes and nickels. If he has four more nickels than dimes, *how many of each coin does he have?*

(*Hint a chart might help and remember to change the dollars into cents to keep units consistent.*)

	Nickels	***Dimes***
Number of	$y + 4$	y
Value of (¢)	$5(y + 4)$	$10y$

Total value of the coins is $1.85 = 185¢

Let y represent the number of dimes that Joe had

∴ The number of nickels is y + 4

$$
\begin{array}{lrcl}
\therefore & 5(y + 4) + 10y & = & 185 \\
 & 5y + 20 + 10y & = & 185 \\
 & 15y + 20 & = & 185 \\
-20) & 15y + 20 - 20 & = & 185 - 20 \\
 & 15y & = & 165 \\
\div 15) & \frac{15y}{15} & = & \frac{165}{15} \\
 & y & = & 11
\end{array}
$$

If y = 11 then…

Number of dimes is y = 11

Number of nickels is $y + 4 = 11 + 4$

$= 15$

Therefore Joe had 11 dimes and 15 nickels.

Check:

Value of the dimes	***=***	***10(11)***
	=	***110***
Value of the nickels	***=***	***5(15)***
	=	***75***
Total value (¢)	***110 + 75 =***	***185 (correct value)***

2. Morgan has three times as many nickels as dimes. Their total value is $1.50. *How many of each coin does she have?*
   (*Hint a chart might help and remember to change the dollars into cents to keep units consistent.*)

	Nickels	***Dimes***
Number of	***3x***	***x***
Value of (¢)	***5(3x)***	***10x***

Total value of the coins is $1.50 = 150¢

Let x represent the number of dimes
∴ The number of nickels is 3x

∴	***5(3x)+ 10x***	***=***	***150***
	15x + 10x	***=***	***150***
	25x	***=***	***150***
÷25)	***$\frac{25x}{25}$***	***=***	***$\frac{150}{25}$***
	x	***=***	***6***

If x = 6 then…
Number of dimes is x = 6
Number of nickels is 3x = 3(6)
= 18

Therefore Morgan had 6 dimes and 18 nickels.

Check:

Value of the dimes	***=***	***10(6)***
	=	***60***
Value of the nickels	***=***	***5(18)***
	=	***90***
Total value (¢)	***60 + 90 =***	***150 (correct value)***

3. Jane has a variety of bills in her purse ranging from one-dollar bills, five-dollar bills, to ten-dollar bills. The total amount in her purse is $43. She has four times as many

one-dollar bills as ten-dollar bills. All together, there are 13 bills in her purse. *How many of each bill does she have?*

	One Dollar	*Five Dollar*	*Ten Dollar*
Number of	***4t***	***13 – 5t***	***t***
Value of ($)	***1(4t)***	***5(13 – 5t)***	***10t***

The total value of the coins is $43

Let t represent the number of ten dollar bills
∴ The number of one dollar bills is 4t

The total number of bills is 13

∴	***4t + t + (number of five dollar bills)***	***=***	***13***
	5t + (number of five dollar bills)	***=***	***13***
-5t)	***5t + - 5t + (number of five dollar bills)***	***=***	***13- 5t***
	number of five dollar bills	***=***	***13- 5t***

Value of the 1 dollar bills + value of the 5 dollar bills + value of the 10 dollar bills = 43

∴	***4t + 5(13 – 5t) + 10t***	***=***	***43***
	4t + 65 – 25t + 10t	***=***	***43***
	65 – 11t	***=***	***43***
-65)	***65 – 11t - 65***	***=***	***43 - 65***
	– 11t	***=***	***-22***
÷-11)	***-11t / -11***	***=***	***-22 / -11***
	t	***=***	***2***

If t = 2 then…

Number of ten dollar bills is	***t***	***=***	***2***
Number of five dollar bills is	***13 – 5t***	***=***	***13 - 5(2)***
		=	***3***
Number of one dollar bills is	***4t***	***=***	***4(2)***
		=	***8***

Therefore Jane has 8 ones, 3 fives, and 2 ten dollar bills.

Check:

Value of the ten dollar bills		***=***	***10(2)***
		=	***20***
Value of the five dollar bills		***=***	***5(3)***
		=	***15***
Value of the one dollar bills		***=***	***1(8)***
		=	***8***
Total value ($)	***20 + 15 + 8***	***=***	***43 (correct value)***

4. Jill is 6 years older than Roberto. In two years, she will be twice as old as Roberto plus one year. *How old is Jill now?*

	Present Age	*Age in 2 years*
Roberto	***y***	***y + 2***
Jill	***y + 6***	***2(y + 2) + 1 or (y + 8)***

There are two ways to write how old Jill will be in 2 years.

Current age plus number of years = two times (Roberts age in 2 years) plus 1

$$y + 8 = 2(y + 2) + 1$$

Solve for y

$$\begin{aligned} \therefore \quad & y + 8 = 2(y + 2) + 1 \\ & y + 8 = 2y + 5 \\ -y) \quad & y - y + 8 = 2y - y + 5 \\ & 8 = 1y + 5 \\ -5) \quad & 8 - 5 = y + 5 - 5 \\ & 3 = y \end{aligned}$$

Roberto is presently 3 years old.

Jill is presently $y + 6 = 3 + 6$
$= 9$ ***years old.***

CHECK:

In two years Roberto will be 5 years old.
In two years Jill will be 9 + 2 which is 11 years old.

Since $2 \times 5 + 1 = 11$ ***the solution is correct.***

5. Maria is 12 years older than Manual. Last year she was twice as old as Manual. *How old are they now?*

	Present Age	*Age last year*
Manual	***y***	***y - 1***
Maria	***y + 12***	***2(y – 1) or y + 11***

There are two ways to write how old Maria was last years.

Current age plus number of years = two times Manual's age

$$y + 11 = 2(y - 1)$$

Solve for y

$$\begin{aligned} \therefore \quad & y + 11 = 2(y - 1) \\ & y + 11 = 2y - 2 \\ -y) \quad & y - y + 11 = 2y - y - 2 \\ & 11 = y - 2 \\ +2) \quad & 11 + 2 = y - 2 + 2 \\ & 13 = y \end{aligned}$$

Manual is presently 13 years old.

Maria is presently $y + 12 = 13 + 12$

$= 25$ ***years old.***

CHECK:

Last year Manual`s age was $y - 1 = 13 - 1$

$= 12$

Since last year Maria (24) was twice as old as Manual (12), the solution is correct.

6. Dan is four times as old as his brother. Four years from now he will be twice as old as his brother. *How old is Dan's brother now?*

	Present Age	***Age in four year***
brother	y	$y + 4$
Dan	$4y$	$4y + 4$ ***or*** $2(y + 4)$

There are two ways to write how old Dan will be in 4 years.

Current age plus number of years = two times (brothers age in 4 years)

$$4y + 4 = 2(y + 4)$$

Solve for y

$$\begin{aligned} \therefore \quad & 4y + 4 = 2(y + 4) \\ & 4y + 4 = 2y + 8 \\ -2y) \quad & 4y - 2y + 4 = 2y - 2y + 8 \\ & 2y + 4 = 8 \\ -4) \quad & 2y + 4 - 4 = 8 - 4 \end{aligned}$$

$$\begin{array}{rrcl} & 2y & = & 4 \\ \div 2) & \frac{2y}{2} & = & \frac{4}{2} \\ & y & = & 2 \end{array}$$

Dan's brother is presently 2 years old.

Dan is presently $4y = 4\,(2)$
$= 8$ ***years old.***

CHECK:

Calculate Dan's age in four years: $4y + 4 = 4(2) + 4$
$= 12$

The brother's age in four years is $y + 4 = 2 \times 4$
$= 6$

Since in 4 years, Dan (12) will be twice his brother`s age (6) the solution is correct.

7. Three friends have a total age of 64 years. The oldest is six years older than the youngest and two years older than the second. *Find each of their ages.*

Let the youngest friend be x years old.

Oldest friend is $x + 6$

Middle friend is $x + 4$

$$\begin{array}{rrcl} \therefore & x + x + 6 + x + 4 & = & 64 \\ & 3x + 10 & = & 64 \\ -10) & 3x + 10 - 10 & = & 64 - 10 \\ & 3x & = & 54 \\ \div 3) & \frac{3x}{3} & = & \frac{54}{3} \\ & x & = & 18 \end{array}$$

$\therefore$ ***The youngest friend is 18 years old.***

The oldest friend is $x + 6 = 18 + 6$
$= 24$ ***years old.***

The middle friend is $x + 4 = 18 + 4$
$= 22$ ***years old.***

Check:

Youngest + middle + oldest $= 18 + 22 + 24$

$= \quad 64$ *(correct total)*

8. Juan has \$1.35 in quarters and pennies. There are 15 coins in all.
How many of each coin does he have?

	Pennies	*Quarters*
Number of	*15- x*	*x*
Value of (¢)	*1(15 - x)*	*25x*

There are 15 coins
Let x represent the number of quarters, therefore there are 15 – x pennies
Total value of the coins is \$1.35 = 135¢

$$\begin{aligned}
\therefore \quad 1(15 - x) + 25x &= 135 \\
15 + 24x &= 135 \\
-15) \quad 15 + -15 + 24x &= 135 - 15 \\
24x &= 120 \\
\div 24) \quad \frac{24x}{24} &= \frac{120}{24} \\
x &= 5
\end{aligned}$$

If x = 5 then…

Number of quarters is $x = 5$

Number of pennies is $15 - x = 15 - 5 = 10$

Check:

Value of the quarters	*25(x)*	*which is*	*25(5) = 125*	
Value of the pennies	*1(15 - x)*	*which is*	*1(10) = 10*	
Total value (¢)			*135*	*(correct value)*

9. Jeannette has \$1.15 in nickels and dimes. If the number of dimes is one less than twice the number of nickels, *find the number of nickels.*

	Nickels	*Dimes*
Number of	*n*	*2n – 1*
Value of (¢)	*5(n)*	*10(2n-1)*

Let n represent the number of nickels, therefore there are 2n – 1 dimes.
Total value of the coins is \$1.15 = 115¢

$$\begin{aligned}
\therefore \quad 5n + 10(2n - 1) &= 115 \\
5n + 20n - 10 &= 115 \\
25n - 10 &= 115 \\
+10) \quad 25n - 10 + 10 &= 115 + 10 \\
25n &= 125 \\
\div 25) \quad \frac{25n}{25} &= \frac{125}{25}
\end{aligned}$$

$$\frac{25}{n} = \frac{25}{5}$$

If n = 5 then…

Number of nickels is $n = 5$
Number of dimes is $2n - 1 = 2(5) - 1$
$= 9$

Therefore Jeannette has 5 nickels and 9 dimes.

Check:
Value of the nickels 5(n) is 5(5) = 25
Value of the dimes 10 (2n-1) is 10(9) = 90
Total value (¢) 115 (correct value)

10. A father is three times as old as his son. Six years ago, he was five times as old as his son. *How old is the father now?*

	Present Age	***Age six years ago***
son	***x***	***x - 6***
father	***3x***	***3x - 6 or 5(x - 6)***

There are two ways to write how old the father is now.

Current age minus number of years = Five times as old as son

$3x - 6 = 5(x - 6)$

Solve for x:

$$\begin{aligned}
3x - 6 &= 5(x - 6) \\
3x - 6 &= 5x - 30 \\
-3x) \quad 3x - 3x - 6 &= 5x - 3x - 30 \\
-6 &= 2x - 30 \\
+30) \quad -6 + 30 &= 2x - 30 + 30 \\
24 &= 2x \\
\div 2) \quad \frac{24}{2} &= \frac{2x}{2} \\
12 &= x
\end{aligned}$$

The son is presently 12 years old.
His Father is presently 3x = 3 (1 2) = 36 years old.
CHECK:

Calculate father's age six years ago: 3x – 6 = 3(12) – 6
$= 36 - 6$
$= 30$

Six years ago the son was = (x – 6)

$= 12 - 6)$
$= 6$

Six years ago the father (30) was 5 times the son`s age (6). Therefore the solution is correct.

11. Crispan and Joan are selling grapefruits. Customers can buy small boxes of grapefruits and large boxes of grapefruits. Crispan sold 3 small boxes of grapefruits and 14 large boxes of grapefruits for a total of $192. Joan sold 11 small boxes of grapefruits and 11 large boxes of grapefruits for a total of $220. *Find the cost each of one small box of oranges and one large box of oranges.*

Let x represent the cost of a small box (in dollars)
Let y represent the cost of a large box (in dollars)

	Number of Small Boxes	Number of Large Boxes	Total Sales Value
Crispan	3	14	$3x + 14y = \$192$ (1)
Joan	11	11	$11x + 11y = \$220$ (2)

Use the "Total Sales Value" equations (1) and (2) to solve for x and y

$$\begin{aligned} & 3x + 14y = 192 && (1) \\ & 11x + 11y = 220 && (2) \end{aligned}$$

$$\begin{aligned} (1) \times 11 \quad & 33x + 154y = 2112 && (3) \\ (2) \times -3 \quad & -33x - 33y = -660 && (4) \end{aligned}$$

$$\begin{aligned} (3) + (4) \quad & 121y = 1452 \\ \div 121) \quad & \frac{121y}{121} = \frac{1452}{121} \\ & y = 12 \end{aligned}$$

Substitute $y = 12$ into (1)

$$\begin{aligned} & 3x + 14y = 192 \\ \therefore \quad & 3x + 14(12) = 192 \\ & 3x + 168 = 192 \\ -168) \quad & 3x + 168 - 168 = 192 - 168 \\ & 3x = 24 \\ \div 3) \quad & \frac{3x}{3} = \frac{24}{3} \\ & x = 8 \end{aligned}$$

Therefore the cost of a small box (in dollars) is $8.00 and the cost of a large box (in dollars) is $12.00.

Check:

Substitute the cost for the small box $x = 8$ and the cost of the large box $y = 12$ into equation (1)

$$\begin{aligned} 3x + 14y &= 192 \\ 3(8) + 14(12) &= 192 \\ 24 + 168 &= 192 \end{aligned}$$

192 = 192
L.S. =R.S. Therefore the solution is correct.

Challenge:

> Interesting Fact:
>
> Coffee is the second most traded commodity in the world. Oil is the first.
>
> http://www.gomestic.com/Consumer-Information/25-Facts-About-Coffee.42195

12. A coffee café specialty coffee is a mix of coffee blends. A 100 pound order of the specialty coffee uses a mix of some light, medium and dark blends. Twice as much was added from the medium blend than from the light blend. The medium blend was 30 pounds. *How much of the dark blend was used in the 100 pound mix?*

Let x represent the amount of light blend
Then the amount of medium blend is twice the amount of light blend = 2x
The medium blend is 30 pounds.

$$\begin{aligned} \therefore \quad & 2x = 30 \\ \div 2) \quad & \frac{2x}{2} = \frac{30}{2} \\ & x = 15 \end{aligned}$$

The blend total is 100lb

Let y represent the amount of dark blend

$$\begin{aligned} \therefore \quad & x + 2x + y = 100 \\ & 15 + 30 + y = 100 \\ & 45 + y = 100 \\ -45) \quad & 45 - 45 + y = 100 - 45 \\ & y = 55 \end{aligned}$$

Therefore the amount of dark blend used in the specialty coffee is 55 pounds.